THE JOURNAL OF FINANCE • VOL. XLVI, NO. 4 • SEPTEMBER 1991

Liquidity, Maturity, and the Yields on U.S. Treasury Securities

YAKOV AMIHUD and HAIM MENDELSON*

ABSTRACT

The effects of asset liquidity on expected returns for assets with infinite maturities (stocks) are examined for bonds (Treasury notes and bills with matched maturities of less than 6 months). The yield to maturity is higher on notes, which have lower liquidity. The yield differential between notes and bills is a decreasing and convex function of the time to maturity. The results provide a robust confirmation of the liquidity effect in asset pricing.

THIS PAPER STUDIES EMPIRICALLY the effects of the liquidity of capital assets on their prices. Amihud and Mendelson (1986, 1989) proposed that liquidity affects asset prices because investors require a compensation for bearing transaction costs. Transaction costs—paid whenever the asset is traded—form a sequence of cash outflows. The discounted value of this cost stream proxies for the value loss due to illiquidity, which lowers the asset's value for any given cash flow that the asset generates.[1] As a result, the return on assets should be an increasing function of their illiquidity (other things equal). For stocks, the illiquidity effect is expected to be strong because their transaction cost sequence is infinite. Amihud and Mendelson (1986, 1989) demonstrated that common stocks with lower liquidity yielded significantly higher average returns, after controlling for risk and for other factors.

These results on the importance of liquidity in the pricing of stocks raise additional questions: (i) does the liquidity effect depend on the specific controls used by Amihud and Mendelson (1986, 1989)? (ii) does illiquidity have a similarly strong effect on the pricing of bonds, whose maturities are finite? and (iii) if liquidity affects bond yields, how is this effect related to the bond's time to maturity?

*Graduate School of Business, New York University, New York, and Faculty of Management, Tel Aviv University, Tel Aviv; and Graduate School of Business, Stanford University, Stanford, CA; respectively. This paper was first drafted while the second author was with The William E. Simon Graduate School of Business Administration, University of Rochester, Rochester, NY. The authors thank Kenneth Garbade, William Silber, and an anonymous referee for valuable suggestions and First Boston Corporation for providing the data.

[1]The impact of the cost of transacting on price can be illustrated as follows (Amihud and Mendelson (1988a)): consider a stock which is expected to be traded once a year at a cost of 1 cent on the dollar value; discounting the infinite stream of transaction costs, at say 8%, gives a present value of 12.5 cents per dollar of value, which is the loss due to transaction costs or the discounted cost of illiquidity.

These questions are answered in this empirical examination of the effects of liquidity on the pricing of U.S. government securities. Specifically, we compare the yields on short-term U.S. Treasury notes and bills with the *same* maturities of 6 months or less. For these maturities, both securities are similar short-term single-payment (discount) instruments generating the same underlying cash flows and having identical risk. Their liquidity, however, is different. This enables us to estimate the effects of liquidity on asset values with no need to control for other factors that affect them. For example, we do not have to assume any capital asset pricing model of the risk-return relation in order to control for risk. Furthermore, unlike the case of stocks that do not have a fixed maturity date, the securities we study here have finite maturities, which enables us to study the interaction between the time to maturity and the yield differential between notes and bills.

In what follows, we briefly describe the features of the U.S. Treasury securities market that are relevant to our study, focusing on the differences in liquidity between securities (Section I). The effects of these liquidity differences are tested on actual dealers' price quotes on bills and notes (Section II). We close with concluding remarks (Section III).

I. Liquidity and the U.S. Government Securities Market

Liquidity in the U.S. government securities market is provided by dealers (or market-makers) and brokers. Dealers trade with retail customers and with each other, standing ready to buy and sell for their own account at their quoted bid and ask prices, respectively. Sellers can execute their orders instantaneously by selling to a dealer at the quoted bid price, and buyers can obtain immediate execution by buying from the dealer at the quoted ask price.[2] Most interdealer trading and much of the retail trading are done through brokers. They have quotation systems which display the dealers' quotes, easing communication and facilitating execution. Dealers and brokers thus provide liquidity services that save investors the costs, risks, and delays of searching for compatible trading partners. As compensation for providing these services, dealers charge investors the spread between the bid and the ask prices, and brokers charge additional fees. Naturally, lower fees and bid-ask spreads are associated with greater liquidity.

U.S. Treasury notes and bills are distinct fixed-income securities. Bills are short-term discount bonds whereas notes are coupon-bearing bonds with far longer maturities. Within 6 months to maturity, however, notes become short-term single-payment securities like bills. Still, notes and bills remain

[2] For a detailed description of the government securities market, see Stigum (1983), Ch. 13. Garbade and Silber (1976) derived the pricing policies of dealers in the government securities market and estimated the determinants of the bid-ask spread. See also Tanner and Kochin (1971) and Garbade and Rosey (1977).

distinguishable with different procedures for yield calculation, quotation, and trading, and their quotes are transmitted on different systems. Traders usually specialize in one type of these government securities,[3] and there are differences between the two markets.

The market for U.S. Treasury bills is substantially more liquid than the market for notes. While a trade of $100 million in treasury bills can be effected almost instantaneously, the situation is quite different for notes of short maturities. Despite the large size of U.S. treasury note issues (typically two to ten billion dollars in our sample), by the time they approach maturity the notes have already been *locked away* in investors' portfolios, and a large part of each issue is not readily available for trading. Investors who wish to trade a large quantity of notes of short maturities have to go through considerable search in order to arrange the quantity desired, imposing an additional fixed cost of search.[4] The differences in liquidity are evidenced by the differences in the bid-ask spread, the brokerage fees, and the standard size of a transaction. The brokerage fee for bills is between $12.5 and $25 per $1 million, compared with $78.125 per $1 million for notes (paid by the party initiating the transaction),[5] and the typical bid-ask spread on bills is of an order of 1/128 of a point compared with 1/32 on notes (both per $100 face value). It is worth noting that this difference in bid-ask spreads cannot be attributed to the often-assumed problem of asymmetric information about fundamental values faced by market-makers because both instruments are affected by the same information. The difference in the spread represents transaction costs borne by dealers when trading notes because of the associated direct and inventory-related costs. Still, notes are far more liquid than stocks. For example, the spread on the highly-liquid IBM stock is about four times larger than the spread on short-term notes.

Treasury notes and bills with less than 6 months to maturity are thus financial instruments with *identical* underlying cash flows but with *different liquidity*, reflected in differences in their transaction costs. Both the bid-ask spread and the brokerage fees are higher for notes compared to bills. The question that arises is how these liquidity differences affect the valuation of notes and bills.

The effect of illiquidity costs is that investors are willing to pay less for the less-liquid security to compensate for its higher transaction costs. We thus obtain the following hypothesis:

The bills, which have lower transaction costs, will have a lower yield to maturity than notes.

Empirical tests of the liquidity effects are presented in the next section.

[3]Some investors may specialize in treasury bills because of regulation or by their own policies. Yet, there are dealers and investors who can freely choose to invest in either instrument.

[4]Garbade and Silber (1976) analyzed the cost of search as a component of the cost of illiquidity in the government securities market.

[5]Stigum (1983), p. 437; costs might have slightly declined since.

II. Empirical Tests

A. The Data

The data were obtained from the quote sheets of First Boston Corporation, a primary dealer in U.S. government securities, and include bid-ask quotes, maturities, and the notes' coupon rates. The quotes are for standard size transactions as of 3:00 P.M. in each trading day. These are *real* quotes that were *pulled off the screen*.[6] Our sample consists of 37 trading days that represent about 5 days in each month between April and November 1987. We included in our sample only bills and notes with less than 6 months to maturity. For these maturities, notes have only one coupon left to be paid at maturity, and thus they become pure discount securities, just as Treasury bills are. Then, the duration and maturity are the same for both securities, and the tax consequences are also the same.

Consider a note n with maturity date τ_n and annual coupon rate C_n, paid semiannually. The quote sheet for day t includes the ask price on the note, P_{tn}, and the bid price Pb_{tn}, in units of 1/32. The actual price paid on the settlement date (typically 2 business days after the transaction date) includes the interest accrued on the note. Let Δt be the number of days from the last coupon payment to the settlement date, and let H be the number of days in the half-year coupon period in which the settlement takes place. The accrued interest on the note is then given by

$$AC = \frac{C}{2} \cdot \frac{\Delta t}{H},$$

and the price actually paid by the buyer is $P + AC$ (we suppress the date and note indices (t, n) whenever they are unambiguous). The time to maturity T $(= T_{tn})$ is the number of days from the settlement date to the maturity date τ. The bid-ask spread on the note relative to the ask price is given by

$$S = \frac{\text{ask price} - \text{bid price}}{\text{ask price} + \text{accrued interest}} = \frac{P - Pb}{P + C\Delta t/2H}.$$

Next consider the bills. The quotes on bills are in terms of discount rates relative to face value, which we converted into price quotes using the formula (Stigum (1983) pp. 46–49)

$$P = 100\left[1 - \frac{T \cdot D}{360}\right], \tag{1}$$

where P is the price, D is the discount rate, and T is the time to maturity (i.e., the number of days from the settlement date to the maturity date of the bill, τ_B). The bill quotes consist of a bid discount D_b and an ask discount D_a,

[6] Being real rather than *representative* quotes, the bid-ask spreads are often substantially narrower than those reported in the *Wall Street Journal*. There, quotes often represent an indication of the price range (particularly for notes).

which we converted into bid and ask prices, Pb and P, using equation (1). Clearly, for bills, $C = AC = 0$.

For each security in our sample, we calculated the (annualized) yield to maturity Y relative to the *ask* price by solving for Y from the following equation:[7]

$$P + AC = \frac{{}^1/_2 C + 100}{(1+Y)^{T/365}}. \tag{2}$$

Notes usually mature either in the middle of the month—the 15th—or at month-end—the 30th or 31st (28th or 29th in February). Bills are auctioned every week and have more frequent maturity dates. For each day in our sample we had a number of notes with maturities of up to 6 months. We matched each note with two bills whose maturity dates immediately straddle the note's. Thus, for each day t in our sample ($t = 1, 2, \cdots, 37$) we assigned to each note N_{tn} with a price quote on that day (n indexes the notes within day t) the two bills $B1_{tn}$ and $B2_{tn}$ with the nearest maturities, satisfying

$$\tau_{B1_{tn}} \le \tau_{N_{tn}} \le \tau_{B2_{tn}}.$$

Thus, the days to maturity T are related by

$$T_{B1_{tn}} \le T_{N_{tn}} \le T_{B2_{tn}}.$$

Finally, we constructed from the pair of bills that match each note the weighted average yield Y_B defined by

$$Y_B = w_1 \cdot Y_{B1} + w_2 \cdot Y_{B2}$$

where $w_1 = (T_{B2} - T_N)/(T_{B2} - T_{B1})$ and $w_2 = 1 - w_1$. In the cases where $T_{B1} = T_{B2}$, $w_1 = w_2 = {}^1/_2$. The yield differential between notes and bills is

$$\Delta Y_{tn} = Y_{N_{tn}} - Y_{B_{tn}},$$

with $n = 1, 2, \cdots, M_t$, M_t being the number of notes in day t, and $t = 1, 2, \cdots, 37$. We sometimes had more than one note with the same maturity because they may have been originally issued at different times for different maturities. In these cases, each note was treated as a separate observation. Altogether, we had 489 matched triplets, each consisting of one note and the two straddling bills, with the note maturities stretching from 9 to 182 days.

B. The Liquidity Effect

Table I presents summary statistics for the number of days to maturity T, the relative bid-ask spread S, and the yield to maturity Y for the 489

[7] The conventional bond yields for notes and equivalent bond yields for bills, included in the quote sheets, are calculated as $\frac{365 \cdot D}{360 - T \cdot D}$ using the *ask* discount and, hence, constitute linear approximations for the actual yield to maturity. Garbade (1983) discussed the bias in this method of yield calculation. We replicated our estimations using these yields, and the results were similar.

Table I

Estimated Means and Standard Deviations for the Days to Maturity T, Relative Bid-Ask Spread S (in %), and Annualized Yield to Maturity Y (in %), for 489 Matched Triplets of Notes and Bills. Each Note Is Matched with Two Bills Whose Maturities Straddle the Note's

The relative bid-ask spread is defined by $S = \frac{\text{ask price} - \text{bid price}}{\text{ask price} + \text{accrued interest}}$, where the accrued interest for bills is zero. The yield to maturity Y is obtained from the equation

$$\text{Ask Price} + \text{Accrued Interest} = \frac{{}^{1}/_{2}C + 100}{(1+Y)^{T/365}},$$

where C is the coupon rate (zero for bills). The differences shown are between the bid-ask spreads S and the yields to maturity Y of notes and the weighted averages of the two matching bills. The data represent 37 days from April to November 1987.

	Days to maturity, T	Spread (%), S	Annual yield (%), Y
Notes (N)			
Mean	97.41	0.0303	6.523
StDev	51.44	0.0004	0.606
Bill 1 ($B1$)			
Mean	94.69	0.00761	6.039
StDev	51.53	0.00547	0.756
Bill 2 ($B2$)			
Mean	100.95	0.00801	6.137
StDev	51.79	0.00664	0.677
Difference (Notes − Bills)			
Mean		0.02252	0.428
StDev		0.00523	0.475

matched triplets of notes and bills. Since the yield curve was generally increasing during the period, we have in general $Y_{B1} < Y_{B2}$. Table I shows two key differences between the bills and the notes:

1) The relative bid-ask spread on notes is greater than that on bills by a factor of about 4. This indicates the lower liquidity of notes compared to bills.
2) The yield to maturity on notes is higher than the yield on bills with the same maturity. On average, $\Delta Y = 0.428\%$ per annum with a standard error of 0.021 and is significantly different from zero.

These observations support the hypothesis that asset returns are a function of liquidity (holding other things constant): the lower the liquidity, the higher the yield measured relative to the ask price.

Given that both instruments also incur different brokerage fees, we recalculated the yields, adding to the ask price the respective costs, viz., \$1/128 per \$100 of face value for notes and \$12.5 per \$1,000,000 of face value for bills. These yield calculations are based on buying and holding to maturity both the note and the bill. The average difference between the resulting yields to maturity was 0.388% per annum (standard error of 0.021, significant at better than 10^{-9}; the yield differential was higher after the October 19, 1987 stock market crash than before it). We thus find that after accounting for both the bid-ask spread and the brokerage fees, the yield to maturity on notes is higher than the yield on bills with the same maturity. This implies that investors are willing to pay a yield concession for the *option* to liquidate their holdings before maturity at lower costs. This is consistent with the results of Amihud and Mendelson (1986, 1989) that the risk-adjusted return on NYSE stocks was an increasing function of the bid-ask spread. But, unlike the case of stocks, no adjustment for risk is necessary here because the two assets we examine have identical underlying cash flows and they differ only in their liquidity. Thus, the yield differential can be traced directly to the liquidity differential.[8]

C. Maturity Effects

Stocks have infinite maturity, and therefore investors must incur transaction costs whenever they liquidate their positions. For finite-maturity securities, investors can receive the redemption value at maturity without incurring liquidation costs. If investors' holding periods—the time until they need to liquidate their investments—were certain and if they could always exactly match the maturity of the notes and bills they buy with their investment horizons, then the yields to maturity on both securities should be the same when calculated from the ask (buying) price and after adjusting the yields for brokerage fees. We found, however, that the yields thus calculated were higher on notes than on bills.

Our evidence thus suggests that there is no perfect matching between investors' planning horizons and securities' maturities. Instead, investors expect that, with a positive probability, a need may arise to sell securities before maturity, at which time they will incur additional transaction costs. It follows that even a finite-maturity security is likely to incur transaction costs through its life, and that these costs could be incurred repeatedly because each buyer has these expectations anew. These transactions costs are higher on notes than they are on bills.

In order to reduce the likelihood of having to sell before maturity and thus save on liquidation costs, an investor could buy a string of short-term securities maturing in sequence. However, this policy makes the investor incur repeated costs of reinvestment (partly fixed) because his horizon is longer than the securities' maturities. Again, these reinvestment costs are higher for notes than they are for bills.

It follows that an investor with an uncertain investment horizon faces a tradeoff between buying short-maturity securities, which may force him to

[8]A similar observation for notes and bills was made by Garbade (1984).

pay transaction costs when reinvesting at their maturity, and long-maturity securities, which may result in the payment of transaction costs when selling before maturity. This tradeoff suggests an interplay between the time to maturity and the yield differential between notes and bills.

To estimate the relation between the time to maturity and the notes' excess yield ΔY_{tn}, we divided the data into 11 groups based on the time to maturity, with each 15 days constituting a group. Each group i ($i = 1, \cdots, 11$) includes the notes and matched bills with $15i < T_{N_{tn}} \leq 15(i+1)$ (the last group includes up to 182 days to maturity).[9] Each maturity group i was assigned a maturity dummy-variable DM_i defined by

$$DM_i = \begin{cases} 1 & \text{if } \ 15i < T_{N_{tn}} \leq 15(i+1) \\ 0 & \text{otherwise} \end{cases}$$

for $i = 1, 2, \cdots, 11$. The estimation of the relation between the yield differential ΔY and the 11 maturity groups employed the pooled time-series and cross-section model

$$\Delta Y_{tn} = a_0 + \sum_{i=1}^{10} a_i \cdot DM_i + b \cdot C_{tn} + \sum_{t=1}^{36} c_t \cdot DD_t + \epsilon_{tn}. \tag{3}$$

The coefficients a_i ($i = 1, 2, \cdots, 10$) measure the difference in the yield differential between the ith maturity group and that of group 11, which has the longest maturity (over 165 days). The model includes the note's coupon rate C_{tn} because it may affect the demand for notes. Institutions that are constrained to distribute only accrued interest (but not the principal) on their holdings may prefer notes with higher coupon rates,[10] and such notes should have correspondingly lower yields, implying that $b < 0$. The dummy variables DD_t, defined as

$$DD_t = \begin{cases} 1 & \text{if the data are from day } t,\ t = 1, 2, \cdots, 36, \\ 0 & \text{otherwise,} \end{cases}$$

control for shifts in ΔY for different days in our sample.

The residual series ϵ_{tn} is subject to heteroskedasticity in the pooled time-series and cross-section estimation. In fact, the residual variances differed across the days in the sample (as might be expected) and as a function of the maturity groups and the coupon. Following Judge et al. (1982), pp. 416–420, we assumed that the residual variances σ_{tn}^2 have the following exponential form:[11]

$$E[\epsilon_{tn}^2] = \sigma_{nt}^2 = \exp\left[k_0 + \sum_{i=1}^{10} k_i \cdot DM_i + l \cdot C_{tn} + \sum_{t=1}^{36} m_t \cdot DD_t\right]. \tag{4}$$

[9] We excluded the notes with up to 15 days to maturity, whose trading is particularly thin (Stigum (1983), pp. 447–449) and thus exhibit erratic behavior. Including this group did not change the essence of the results.

[10] This was suggested to us by Kenneth Garbade.

[11] This form guarantees positive estimated variances (Judge et al. (1982), p. 416).

The corresponding estimation procedure was as follows. We first estimated model (3) and used its residuals to estimate model (4). The estimated variances from this model were then used for a GLS estimation of model (3). A test of the resulting model for serial correlation could not reject the hypothesis of no serial correlation (DW = 1.83, inconclusive).

The estimation results are presented in Table II. In Panel A the estimation model includes the group-dummy variables (as well as the day-dummies) and excludes the coupon variable.[12] Panel B presents the results for the complete model (3). The results are similar under both specifications. The coefficients a_i, which measure the excess yield in each group i relative to the longest-maturity group (group 11), are decreasing and convex in i. That is, the yield differential ΔY is a decreasing and convex function of the time to maturity. For example, in the third column the note-bill yield difference (relative to that of group 11, which had over 166 days to maturity) declines from 46.4 basis points for the first maturity group (16–30 days to maturity) to 22.7 basis points for the second maturity group, a fairly steep decline. The decline in the yield differential for higher maturity groups becomes gradually more moderate. The pattern of a_i closely fits the function $1/(i + {}^1/_2)$,[13] with the correlation between them being 0.97. Thus, the yield differential ΔY seems to conform closely to a linear function of the reciprocal of the time to maturity. The coefficient of the coupon variable, -0.014, implies that a 100-basis-point difference in the coupon rate is associated with a 1.4-basis-point difference in the note-bill yield differential.

We then estimated directly the functional relation between the yield differential and the time to maturity, assuming that ΔY is a linear function of the reciprocal of the days to maturity T. The estimation results were as follows:

$$\Delta Y_{tn} = \gamma_0 + \underset{(9.47)}{12.03} \cdot (1/T_{N_{tn}}) - \underset{(3.23)}{0.014} \cdot C_{tn} + \sum_{t=1}^{36} \gamma_t \cdot DD_t + \epsilon_{tn}, \quad (5)$$

(t-values in parentheses; $DW = 1.85$). The estimation employed the GLS procedure used to estimate model (3) with the variance structure (4). By model (5), increasing the time to maturity from 30 days to 150 days reduces the note-bill yield difference by 32.1 basis points. Notably, the coefficient of $(1/T_{N_{tn}})$ was greater after the October 1987 crash and smaller before it. The coefficient of the coupon rate remains unchanged.[14]

[12] For this model, we had $DW = 1.86$, inconclusive.

[13] Recall that group i corresponds to an average maturity of $15(i + {}^1/_2)$ days.

[14] We also employed another methodology to estimate the relation between the yield differential and the days to maturity. Instead of a pooled time-series cross-section estimation, we estimated model (5) for each day t, $t = 1, 2, \cdots, 37$, (without the day-dummies) and obtained 37 estimates for the coefficients of $1/T$ and C. The means of these coefficients were similar to those estimated in (5), and they were highly significant.

Table II

Coefficients of the Maturity Dummy Variables and of the Coupon Rate in the Pooled Time-Series and Cross-Section Regression (3) (t-Values are in Parentheses)

Panel A:

$$\Delta Y_{tn} = a_0 + \sum_{i=1}^{10} a_i \cdot DM_i + \sum_{t=1}^{36} c_t \cdot DD_t + \epsilon_{tn},$$

where ΔY_{tn} is the yield differential between notes and bills with matched maturities, and DM_i are dummy variables for the 11 maturity groups, defined by

$$DM_i = \begin{cases} 1 & \text{if } 15i < T_{N_{tn}} \le 15(i+1), \\ 0 & \text{otherwise,} \end{cases}$$

for $i = 1, 2, \cdots, 11$. $T_{N_{tn}}$ is the number of days to maturity for note n on day t. The coefficient a_i ($i = 1, 2, \cdots, 10$) measures the yield differential between maturity group i and the 11th maturity group. DD_t are the 36 day-dummy variables, $DD_t = 1$ if the observation is on day t and 0 otherwise.

Panel B:

$$\Delta Y_{tn} = a_0 + \sum_{i=1}^{10} a_i \cdot DM_i + b \cdot C_{tn} + \sum_{t=1}^{36} c_t \cdot DD_t + \epsilon_{tn},$$

where C_{tn} is the coupon rate (in %) for note n on day t.

The sample includes 466 triplets of notes and bills of matched maturities for 37 days in 1987. All estimations use GLS with the variance function

$$\sigma_{nt}^2 = \exp\left[k_0 + \sum_{i=1}^{10} k_i \cdot DM_i + l \cdot C_{tn} + \sum_{t=1}^{36} m_t \cdot DD_t\right].$$

Coefficient Estimates		
Variable	Panel A	Panel B
Constant	0.662	0.765
	(5.30)	(6.33)
DM_1	0.443	0.464
	(6.50)	(6.54)
DM_2	0.253	0.227
	(5.27)	(4.73)
DM_3	0.166	0.141
	(4.19)	(3.43)
DM_4	0.114	0.107
	(3.25)	(3.24)
DM_5	0.075	0.066
	(2.53)	(2.51)
DM_6	0.055	0.058
	(1.49)	(1.90)
DM_7	0.059	0.040
	(1.90)	(1.45)
DM_8	0.123	0.121
	(3.64)	(3.67)

Table II—*Continued*

Variable	Panel A	Panel B
DM_9	0.032	0.051
	(0.87)	(1.63)
DM_{10}	0.049	0.029
	(1.13)	(0.70)
Coupon		−0.014
		(3.20)

The estimates of model (5) establish the existence of a relation between the notes' excess yield over bills and their maturity, which is linear in the reciprocal of the time to maturity. This relation can be reasoned as follows. A longer maturity enables investors to depreciate any fixed transaction costs over a longer time period. Therefore, the excess yield that investors require to compensate for this cost will be lower for longer maturities.

The results of model (5) on the yield differential as a function of the time to maturity directly imply that the price differential between bills and notes may also be a function of maturity. The price differential is defined as

$$\Delta P = P_B - 100 \cdot (P_N + AC)/(100 + C/2),$$

where $P_B = w_1 \cdot P_{B_1} + w_2 \cdot P_{B_2}$ is the weighted average of the prices of the bills whose maturities straddle the note's, and the price of the note is adjusted to have a redemption value (= face value plus half the coupon rate) of 100, as is the case for bills. If the yield differential ΔY were independent of the time to maturity, ΔP would be increasing in the time to maturity. However, we found that ΔY decreases in maturity, which raises the question of how ΔP depends on the time to maturity. From model (5) we obtained that the mean of the intercept terms (plus the mean coupon term) was positive, implying that ΔP is an increasing function of T. To test directly the relation between ΔP and T, we estimated model (3), replacing ΔY by ΔP. The estimation (which employs the GLS procedure as for model (3)) showed that ΔP is an increasing function of the time to maturity.[15]

The price differential between bills and notes suggest the possibility of profitable arbitrage opportunities, especially in long maturity bills and notes, where the price differential is larger. This is analyzed in the next section.

III. Arbitrage Opportunities

The existence of a yield differential between bills and notes may give rise to arbitrage opportunities. Specifically, an apparent profitable arbitrage is

[15] The coefficients of the maturity dummy-variables a_i were (going from short to long maturities) as follows: −0.070, −0.056, −0.057, −0.050, −0.043, −0.042, −0.018, −0.013, +0.003, −0.003, and +0.022, with t-statistics 6.43, 5.91, 5.81, 5.32, 4.95, 4.23, 1.85, 1.18, 0.30, and 1.56, respectively. The coefficients of the maturity-dummy variables are increasing, indicating that ΔP is an increasing function of the time to maturity. The coefficient of the coupon variable was −0.0036 (t = 3.20), and DW = 1.83.

buying a note and selling short a bill of the same maturity and the same redemption value, and then holding this position until maturity, at which time both securities are redeemed. Because of the yield differential, the cost of the long position in the note is lower than the proceeds from shorting a bill of the same maturity. Such arbitrage is *riskless* because the values of both securities at maturity are known with certainty. Hence, in the spirit of our foregoing analysis, it can be evaluated independently of investors' utility functions or specific models of asset pricing.[16]

However, given that the yield differential is created by differences in transaction costs, these costs should be accounted for in evaluating the arbitrage transaction.

First, the note is bought at the ask price, whereas the bill is sold at the bid price, and therefore the cost of transacting should reflect the bid-ask spread.

Second, there are brokerage fees of 1/128 point, or $78.125 per $1 million for notes and brokerage fees between $12.5 and $25 per $1 million for bills (Stigum (1983), p. 437). The fees are paid by the party that initiates the transaction.

Third, shorting the bill entails a cost of borrowing it which typically equals 0.5% per annum (Stigum (1983), p. 288); this cost varies between transactions, but the 0.5% is quite representative.

Clearly, an investor who holds neither security and wishes to take a position in this market can exploit the yield differential by buying a note instead of a bill. In Section II. *B*., we found that the *net* yield calculated from the ask price (accounting for the difference in brokerage costs) is still higher for notes than for bills; the mean difference in yields was 0.388% (standard error = 0.021). Thus, an investor who takes a position in this market and holds it to maturity may expect a gain by buying a note instead of a bill.

Next, an investor who currently holds a bill can make an immediate gain by selling the bill and buying instead a note of the same maturity and the same face value. The profit for this investor is calculated by the following two profit measures. The first is

$$PROFIT1 = Pb_B - 100 \cdot (P_N + AC)/(100 + C/2),$$

where Pb_B is the bill's bid price. *PROFIT*1 is the apparent gain from selling a bill at the bid price and buying a note at the ask price in a quantity which would make the note's redemption value equal to 100, the bill's face value. Given the second cost component, the brokerage fees borne by the party initiating the transaction, the gain to the investor who switches from holding a bill to holding a note is given by

$$PROFIT2 = PROFIT1 - (78.125 + 12.5) \cdot 100/1000000.$$

[16] An evaluation of risky arbitrage, where unwinding the position occurs before maturity at uncertain prices, requires modeling assumptions such as knowledge of investors' risk aversion or utility functions; see Tuckman and Vila (1990).

Finally, we calculated the profit to an investor who currently holds neither securities and is constructing a pure arbitrage position:

$$PROFIT3 = PROFIT1 - (78.125 + 12.5) \cdot 100/1000000$$
$$-0.005 \cdot (T_N/365) \cdot PB_B.$$

This measure accounts for the full transaction costs of the arbitrage, viz., the brokerage fees and the cost of borrowing the bill, in addition to the bid-ask spread effect captured by *PROFIT*1. *PROFIT*3 shows the gain made at the time the arbitrage position is constructed. Given the results in Section II.*C*. on the price differential between bills and notes being increasing in the time to maturity, it seems that there are arbitrage profits which are higher for longer maturities. However, the third cost component—the cost of borrowing the bill—is increasing in the time to maturity and thus counteracts the effect of the positive relation between maturity and price differential.

Table III presents the three measures of arbitrage profits for the total sample and for the 11 maturity groups. On average, *PROFIT*1 has a positive value of 7.8 cents per $100 redemption value and *PROFIT*2 has a positive value of 6.9 cents per $100 redemption value. This means that switching from an investment in bills to an investment in notes can be profitable, *provided* the investor is certain that he will not have to liquidate the

Table III

Arbitrage Profits (in Dollars) from Selling Bills and Buying Notes of the Same Maturity. Numbers shown are Profits (in Dollars) per $100 Redemption Value in Each Instrument by Maturity Group

*PROFIT*1 = profit from selling the bill at the bid price and buying the note at the ask price. *PROFIT*2 = *PROFIT*1 minus brokerage costs. *PROFIT*3 = *PROFIT*2 minus the cost of borrowing the bill.

The sample includes 489 triplets of notes and bills of matched maturities for 37 days in 1987.

	*PROFIT*1	*PROFIT*2	*PROFIT*3		
Maturity (days)	Mean	Mean	Mean	Min	Max
All maturities	0.078	0.069	−0.062	−0.565	0.200
9–15	0.013	0.004	−0.012	−0.071	0.044
16–30	0.043	0.034	0.001	−0.072	0.082
31–45	0.055	0.046	−0.008	−0.070	0.136
46–60	0.056	0.047	−0.027	−0.091	0.079
61–75	0.082	0.073	−0.020	−0.088	0.065
76–90	0.070	0.061	−0.052	−0.101	0.044
91–105	0.061	0.052	−0.080	−0.232	0.064
106–120	0.102	0.093	−0.059	−0.181	0.158
121–135	0.102	0.093	−0.078	−0.398	0.200
136–150	0.081	0.072	−0.119	−0.565	0.045
151–165	0.129	0.120	−0.091	−0.417	0.159
Over 165	0.104	0.095	−0.137	−0.467	0.028

investment before it matures. (In fact, this type of investor mitigates the institutional differences between trading notes and bills, thereby reducing the yield differential between them.) However, this profit becomes negative, −6.2 cents on average, after subtracting the cost of borrowing the bills in order to short them. This pattern applies to all maturities except one (of 16–30 days), where *PROFIT*3 is practically zero. These results imply that on average, the yield differential between bills and notes cannot be exploited for profitable arbitrage. Yet, in all maturity groups there were specific cases of apparent opportunities for arbitrage profits, as indicated by the Maximum column under *PROFIT*3. There are a number of possible explanations for this. The calculated profit may not have been feasible because of difficulties in constructing the arbitrage. For example, when a particular bill is *special* (e.g., held to hedge against particular periodic payments), the cost of borrowing it may be higher than the assumed 1/2%. Also, our calculations assumed that the short position in the bills can be held to maturity, and thus no additional cost is incurred by prematurely reversing the position. A short position where the lender of the security cannot call it back for a fixed term may sometimes be more costly, because the lender loses liquidity. Alternatively, the cases with positive profit could indicate transitory disturbances in the instruments' prices, which could indeed be profitably exploited by arbitrageurs.[17]

IV. Concluding Remarks

If liquidity affects the pricing of capital assets, then both financial theory and practice should incorporate the liquidity effect. Investment decisions and portfolio composition should be based on the liquidity of capital assets as well as on their expected return and risk (however measured). In corporate finance, increasing the liquidity of the firm's financial claims should be regarded as a worthwhile objective,[18] and policymakers should assess the impact of regulatory and public policy initiatives by their contribution to market liquidity.[19] Further, trading technology is a major determinant of asset liquidity, and its role should be evaluated in light of its liquidity effect (cf. Amihud and Mendelson (1988b)).[20]

Amihud and Mendelson (1986, 1989) showed that liquidity is an important factor in asset pricing because the expected returns on stocks increase with their illiquidity, measured by the bid-ask spread. The estimated liquidity effect was strong and significant, and it persisted after controlling for system-

[17] Dealers in the government securities market temporarily change their quotes in response to temporary changes in their inventories; see Garbade and Silber (1976).

[18] See Amihud and Mendelson (1988a).

[19] A case in point is the recently proposed securities transaction tax that would reduce liquidity and thus increase the yields required by investors. See Amihud and Mendelson (1990) on the effects of such a tax.

[20] Advances in trading technology may naturally affect the liquidity differential between notes and bills and the corresponding yield differential.

atic (beta) risk, size, and unsystematic risk, controls which depend on the specific theory of the risk-return relation. The question that arises is whether the dependence of expected returns on liquidity is robust (in particular, whether it depends on the specific controls for risk) and whether it applies not only to stocks, where transaction costs are incurred infinitely many times, but also to assets with finite and short maturities.

This study examined the effects of illiquidity on the yields of finite-maturity securities with identical cash flows: U.S. Treasury bills and notes with maturities under 6 months. For these maturities, both securities are effectively discount bonds and should be equivalent. Their liquidity, however, is different: the cost of transacting bills is lower than the cost of transacting notes. This enables us to study the relation between asset yields and liquidity without the need to control for other factors.

We find that (i) the yield to maturity on notes is higher than the yield on bills of the same maturity and (ii) the excess yield on notes over that on bills is a decreasing function of the time to maturity and is approximately linear in the reciprocal of the time to maturity. These results confirm and extend Amihud and Mendelson's (1986, 1989) earlier findings on the existence of a liquidity effect. Here, the matching of cash flows of the two instruments makes the results independent of any particular theory of pricing risk. In conclusion, our results show that asset liquidity is an important factor which must be reckoned with in asset pricing.

REFERENCES

Amihud, Yakov and Haim Mendelson, 1986, Asset pricing and the bid-ask spread, *Journal of Financial Economics* 17, 223–249.

——— and Haim Mendelson, 1988a, Liquidity and asset pricing: Financial management implications, *Financial Management* 17, 5–15.

——— and Haim Mendelson, 1988b, Liquidity, volatility and exchange automation, *Journal of Accounting, Auditing and Finance* 3, 369–395.

——— and Haim Mendelson, 1989, The effects of beta, bid-ask spread, residual risk and size on stock returns, *Journal of Finance* 44, 479–486.

——— and Haim Mendelson, 1990, The effect of a transaction tax on securities returns and values, Working paper, The Mid-America Institute for Public Policy Research.

Garbade, Kenneth, 1983, Invoice prices, cash flows, and yields on treasury bonds, *Topics in Money and Securities Markets*, Bankers Trust Co.

———, 1984, Analyzing the structure of treasury yields: Duration, coupon and liquidity effects, *Topics in Money and Securities Markets*, Bankers Trust Co.

——— and William Silber, 1976, Price dispersion in the government securities market, *Journal of Political Economy* 84, 721–740.

——— and Irene Rosey, 1977, Secular variation in the spread between bid and offer prices on U.S. treasury coupon issues, *Business Economics* 12, 45–49.

Judge, George G., R. Carter Hill, William Griffiths, and Tsoung-Chao Lee, 1982, *Introduction to the Theory and Practice of Econometrics*. (John Wiley & Sons, Somerset, NJ).

Stigum, Marcia, 1983, *The Money Market*, (Dow Jones-Irwin, Homewood, IL).

Tanner, Ernest J. and Levis Kochin, 1971, The determinants of the difference between bid and ask prices on government bonds, *Journal of Business* 44, 375–379.

Tuckman, Bruce and Jean-Luc Vila, 1990, Arbitrage with holding costs: A utility-based approach, Working paper S-90-19, Stern School of Business, New York University.