Dear Ms Hamilton,

Re: Insurance Corporation of British Columbia
2014 Revenue Requirements Application

Further to Commission Order G-155-14 with respect to the above noted Application, please find enclosed my Intervener Evidence.

Kind regards,

Richard T. Landale
Enclosure
cc: Registered Interveners
ICBC 2014 REVENUE REQUIREMENT APPLICATION

INTERVENER EVIDENCE SUBMISSION
By
RICHARD T. LANDALE
INTERVENER C1

TO THE

BRITISH COLUMBIAN UTILITIES COMMISSION
900, HOWE STREET,
VANCOUVER
BRITISH COLUMBIA
V6Z 2N3

IN THE MATTER OF

INSURANCE COMPANY OF BRITISH COLUMBIA
2014 REVENUE REQUIREMENT APPLICATION

JANUARY 12th, 2015
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## EXHIBITS

RTL EVIDENCE Actuarial Weighted MCT Calculators for Bodily Injury
#1-1, #1-2, #1-3, #1-4, #1-5

RTL EVIDENCE Basic Premium Impact Graphs #2.1, #2.2, #2.3
(Data Sheet for #2.1 and #2.2)

RTL EVIDENCE #3-1 Canada Pension Plan Calculations and Federal
Consumer Price Index to December 2014 #3-2

APPENDIX A - Intervener Response C1-4 to ICBC

APPENDIX B - Bornhuetter Ferguson Principle Model

APPENDIX C - LISTING OF VARIOUS OTHER MODELS AND
METHODS

APPENDIX D - ACTUARY CERTIFICATIONS 2006, 2007 and 2014
PREFAE
1. The evidence presented to the British Columbian Utilities Commission (BCUC) for consideration address three topics.
   Actuarial Modeling for Bodily Injuries
   Senior Citizen CPP and the Federal CPI
   Highlights from filed evidence exhibits

INTRODUCTION TO ACTUARIAL MODELING FOR BODILY INJURIES
2. The evidence presented challenges ICBC Exhibit C.1.0 and its related, yet selected Exhibits along with my version of actuarial modeling evidence as they relate to Bodily Injury – Personal, only by way of example, but not to the exclusion of all other basic insurance lines of business / categories and rate class.

3. I unreservedly advise that my actuarial model evidence is a total fabrication. It is a construction to make the point that ICBC also constructs their models to justify and support the end, their end for a 5.2% Indicated Rate Change.

4. I am not an actuary, or an accountant, so I make no claim to assert my evidence “RTL Evidence #1-1, #1-2, #1-3, #1-4, #1-5 as fact, but rather as examples of how one can manufacture an outcome, while maintaining ethical and professional standards, albeit while following Special Direction IC2.

5. To lead into this evidence I remind the Commissioners to review ICBC’s original application exhibits and then ICBC’s errata submission September 18th, document exhibit B-3-1 “Re: Errata for Appendix 11 B of ICBC’s 2014 Revenue Requirements Application”, along with my response dated September 19th, document exhibit C1-4, attached herein as Appendix A. I would also remind the Commissioners that from the 2011 RRA I have challenged ICBC’s actuarial modeling in various submissions, letters to the Commission and during the 2013 RRA oral hearings. To date ICBC has dismissed me as well as the BCUC in their last two decisions in this regard. I learn from each occasion, and will continue to craft my skills, knowledge and experiences.

DISCUSSION of EXHIBIT C.1.0 SUMMARY of BODILY INJURY SELECTIONS – PERSONAL
6. Para 1 should be the alarm bell to the reader, as it opens with the words, quote: “The baseline selection” along with the title of this exhibit. Followed on line 4, quote: “Adjustments to the baseline selection are made when circumstances warrant a departure.” Followed on line 7, quote: “final selections (summarized in Exhibit C.1.1) and departures from the baseline in the May 2104 reserve review”

7. What we have here are three manufactured actuarial selections to create a predetermined outcome, while in the next paragraph ICBC will explain these three selections, example: In para 2 it says: quote: “Exhibits C.1.2.1 to C.1.2.3 – Count Development Method – The baseline in Exhibit C.1.2.2 is selected up to 161 months and a factor of one is selected from 161 months”.

8. The idea given to the reader in Exhibit C.1.0 para 1., was there would be some analysis and discussion (quote: revenue requirements analysis is discussed below). The only meritorious information to be gleamed from para 2 is that ICBC selected 161 months and a factor of one. ICBC provides no analysis or discussion as to the pros and cons for this 161 month selection period. So why not 113 or 173 months or 209 months? Where does the Factor of One come from? ICBC refers the reader to Exhibit C.1.2.1, which provides a listing of Accident Years AY 1997 to 2013 a spread of 209 months, or 17 years plus 5 months ending May 2014. The reader is further referred to
Exhibits C1.2.2 and C1.2.3 for further numerical confusion, or should one say ... selection of data and the actuarial creation of multiply factors. All without analysis or discussion as indicated by ICBC in para 1. referenced above, (see Exhibits C1.2.1 for 2013 and 2014).

9. Critically I must question the BCUC for their prior approval to this kind of actuarial modeling. I went back to my printed volumes to find the same exhibits for the 2013 RRA, please see the reduced copies for 2013 and 2014 respectively below:

### Insurance Corporation of British Columbia

#### Exhibit C.1.2.1

<table>
<thead>
<tr>
<th>Accident Year</th>
<th>Reported Claim Count</th>
<th>Data Notification</th>
<th>Data Adjustment</th>
<th>Adjusted Reported Count</th>
<th>Age-to-Age CDF*</th>
<th>Age-to-Ultimate CDF</th>
<th>Selected Insured Count</th>
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</tr>
</tbody>
</table>

(1) From Exhibit C.1.2.2
(2) From Exhibit C.1.2.3
(3) CDF - Count Development Factor

### Insurance Corporation of British Columbia

#### Exhibit C.1.2.1

<table>
<thead>
<tr>
<th>Accident Year</th>
<th>Reported Claim Count</th>
<th>Data Notification</th>
<th>Data Adjustment</th>
<th>Adjusted Reported Count</th>
<th>Age-to-Age CDF*</th>
<th>Age-to-Ultimate CDF</th>
<th>Selected Insured Count</th>
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<td>705,316</td>
</tr>
</tbody>
</table>

(1) From Exhibit C.1.2.2
(2) From Exhibit C.1.2.3
(3) CDF - Count Development Factor

WHY ARE THESE NUMBERS NOT THE SAME AS REPORTED IN 2013 RRA FOR THE SAME "AY" THE SAME QUESTION APPLIES TO "AY" PERIOD 1999 TO 2011 INCLUSIVE?
10. ICBC offers no explanation for the revisions to Column (1) between 2013 and 2014 in these exhibits, along with the corresponding weighting or multiply factors. So again there can be no independent verification.

11. To continue the criticism, one could surmise that perhaps the numbers are different because ICBC has closed the “CWA” and/or “CWP” for each respective (AY) year, only guessing on my part, since there has been no analysis or discussion. What is relevant about this question is how column 7 is finally computed in each exhibit. Are these numbers correct in light of the foregoing commentary?

12. Assuming the BCUC has approved this model, they also gave license apparently to ICBC actuaries to fiddle with the multiply factors in columns 5 and 6. Which finally confirms my original point, that ICBC actuaries can construct, manipulate figures and multiply factors to develop a desired outcome, all under the guise of accepted actuarial practice?

13. Understanding the product in column 7, namely “38,491” is critically important because Exhibit C1.1.1 AY 2014 column 1 is the “Selected Incurred Count”. If one were to change the “Count Development Factor” (CDF) column 6 in exhibit C.1.2.1 to the same factor used in ICBC 2013 RRA the “Selected Incurred Count” would be 37,587 x 1.0230 = 38,451 rather than the 2014 RRA “Selected Incurred Count” of 38,491. So why then is the value of 50 Selected Incurred Count differential important, because of the upstream/downstream model utilization in Exhibit C.1.1.1

14. As Exhibit C.1.1.1 translates the “Selected Incurred Count” into dollars, namely; Selected Incurred Loss $1,205,628,000, adjusted upward to $1,360,367,000 This exhibit is supported by countless other exhibits that are manifested through the application of the Bornhuetter Ferguson Principle model for loss calculation, see Appendix B for a 26 page explanatory paper. Please see discussion of ICBC Exhibit C.1.0 Para 3, on page 5 paragraph 24 of this evidence. It is impossible for any Intervener to follow ICBC’s Exhibit “C” excel spreadsheets to confirm ICBC data or weighting factors. I know, I spent two days trying, and have no worthy evidence to present for my effort.

So what is “Selected Incurred Loss”, I do not know, the following may suggest an overview:

15. “Loss Reserving
From Wikipedia, the free encyclopedia
Loss Reserving or Outstanding claims reserves refers to the calculation of the required reserves for a tranche of general insurance business.[1]
Typically, the claims reserves represent the money which should be held by the insurer so as to be able to meet all future claims arising from policies currently in force and policies written in the past. Methods of calculating reserves in general insurance are different from those used in life insurance, pensions and health insurance since general insurance contracts are typically of a much shorter duration. Most general insurance contracts are written for a period of one year. Typically there is only one payment of premium at the start of the contract in exchange for coverage over the year. Reserves are calculated differently from contracts of a longer duration with multiple premium payments since there are no future premiums to consider in this case. The reserves are calculated by forecasting future losses from past losses.
The more popular statistical methods in claims reserving are the Chain Ladder Method and the Bornhuetter Ferguson Method.
The Chain Ladder Method uses data in a two dimensional array representing occurrence and development of claims. The upper left of this matrix contains known values (in the past) which are used to estimate the remaining figures (i.e. arising in the future).
The Bornhuetter Ferguson Method is a Bayesian technique. This means that it incorporates both an independently derived prior estimate of ultimate expected losses as well as estimates generated by the same kind of matrix described above. These are weighted by what is called a credibility factor, ideally giving preference to the more reliable projection, but taking both into consideration.”
16. The underlined further reinforces my assertion the ICBC constructs data to for fill their need to raise basic insurance rates by way of their own "prior estimates" using accepted "credibility factor" actuarial practice / judgement. None of which can be independently verifiable from the exhibits filed.

17. This is further illustrated by a search for Bayesian technique. Although the following article discusses filtering techniques for email spam, it illustrates how multiplier factors can be generated from historical database statistics. The analogy in my opinion is quite compelling.

**APPLYING BAYESIAN TECHNIQUE TO FILTERING SPAM**
(http://advat.blogspot.ca/2012/04/applying-bayesian-technique-to.html) extracted quotes from article:

"Introduction

The white paper, "Why Bayesian filtering is the most effective anti-spam technology" describes how a company can apply Bayesian mathematics to spam e-mails' problem by creating an adaptive, 'statistical intelligence' technique that increases spam detection rates. The unique characteristic that distinguishes from other spam filters is that the company or organization can customize the filter based on company or organization’s email characteristics, and update the database with newly detected spam characteristics.

Summary

Bayesian filtering is based on the principle that most events are dependent and that the probability of an event occurring in the future can infer from previous occurrences of that event. According to the paper, before we can filter spam using this method, we need to generate a database with words and token collected from a sample of spam mail and valid mail, also referred to as 'ham'. A probability value is assigned to each word or token, which is based on calculations that take into account how often that word occurs in spam as opposed to legitimate mail (ham). The word probability is calculated by analyzing the users’ outbound mail and by analyzing known spam.

For example: If the word “mortgage” occurs in 400 of 3,000 spam mails and in 5 out of 300 legitimate emails, then its spam probability would be \( \frac{400}{3000} \times \frac{5}{300} = 0.8889 \).

18. In the example above the reader has a clear understanding how the final probability factor is developed. It is by this example I believe ICBC should provide all the necessary information for the BCUC and Interveners to understand how ICBC develops their “factors”. This could be achieved by filing the original excel spreadsheets with cell formulas turned on, (& the Evaluate Formula feature).

19. Returning to ICBC, and how they create the hundreds of multiplier factors is the crux of this evidence. In my five exhibits RTL Evidence #1-1, #1-2, #1-3, #1-4, #1-5 are examples of how choosing a baseline of $200,000,000, then imposing at factor of 1 (100%) or (50%) or (25%), or completely picking a different baseline $300,000,000, has very little impact on the final MCT, which is minimal in spite of the different baseline dollar values. It’s all about selection – token (base), multiply factors.

20. In my example: RTL Evidence #1-1, #1-2, #1-3, #1-4, #1-5 I have selected five categories (token), Bodily Injury (BI), Smartphone, Legal Representation (Legal Rep), Medical Inflation (MED Infltn) and Rate Smoothing. (it is all I could fit on one page), not to the exclusion of meaning to dismiss many other categories that are included in ICBC’s RRA.

21. Then choosing weighting factors (multiply factors) that are totally arbitrary (for example purposes only), which were consistently applied across the four models. So by way of expression, using consistent weighting values from model to model, the changes in the MCT values is only impacted when a second variable is added, the 50% and 25%, the (multiply factors).
22. The 50% and 25% variables are user input data to test for a desired outcome; being to identify the threshold for the CRC refund against the approved 2013 RRA BCUC decision for 160% MCT ceiling, and British Columbians are eligible for the CRC refunding imposed by ICBC on their basic insurance premiums, an informal tax, liken to the Non Refundable Tax Credit for Personal Income Tax returns imposed by the Federal Canada Revenue Agency.

23. Again by way of my example, any database subjected to a user predisposed input can be manipulated to craft the 5.2% Indicated Rate Change for Basic Insurance Premiums, while following accepted actuarial practice standards and procedures.

24. Turning back to ICBC Exhibit C.1.0. Para 3., page 3 paragraph 14 of this evidence quote:

“3. Exhibit C.1.3.1 – Basic Bodily Injury Incurred Loss Summary – Shows that the selected incurred loss for accident years 1997 to 2011 is averaged from the incurred development method and the paid development method; while for accident years 2012 to 2013 it is averaged from the incurred and paid Bornhuetter-Ferguson methods”.

25. How can this paragraph (C.1.3.1 – Basic Bodily Injury Incurred Loss Summary), be described as “revenue requirements analysis are discussed below”?

26. ICBC provides no analysis or little to no discussion, just a statement of two selection methods over different “AY” periods... No more information can be gained by the reader to facilitate comprehension of Exhibit C.1.3.1

27. ICBC Exhibit C.1.0. Para 4 to 12 inclusive; These paragraphs do contribute information, and helped direct attention to specific data. Admittedly, I got lost several times, and I am unable to comment, which should not be construed as agreement or acceptance. Rather than to repeat my refrain regarding manufactured multiplier factors, they are a real disturbance to me, and should be to the BCUC Commissioners.

28. I do believe ICBC should provide a great deal more detailed analysis and discussion across the entire RRA regarding the development of their weighting system and multiplier factors along with the spreadsheet formula already given in various footnotes.

29. Attached to my Intervener Evidence email dated January 12th 2015 to the BCUC Commission Secretary, I have attached my original Excel Spreadsheet (format .xlsx): RTL EVIDENCE #1.1, #1.2, #1.3, #1.4, #1.5, which is submitted as evidence for scrutiny by all parties to this RRA.

FOLLOWING THE MONEY TO THE MCT

Discussion
30. The first point came from the 2014 RRA Review Working Session slides 6 to 13 inclusive, where the words “Capital Maintenance” and “Capital Build” provided the short link to Figure 3.2 Indicated Rate Change line item 6 Capital Maintenance of 0.2% Slide #8 “Example: Maintaining Target of 145%”

<table>
<thead>
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<th>SLIDE 8 EXAMPLE: MAINTAINING TARGET OF 145%</th>
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<tbody>
<tr>
<td>145% = $1,700,000,000.00 x $80,000,000.00</td>
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<td>$1,200,000,000.00 x (1+4.7%)</td>
</tr>
<tr>
<td>145% = $108,245,781.60</td>
</tr>
</tbody>
</table>

31. I know this slide is only an example, but I could not verify the formula in this slide in my review of the application. By starting on page 3-28 of the RRA
D.3.2 REQUIRED PREMIUM CALCULATION

74. Required premium is calculated using a discounted cash flow method, as was the case in prior revenue requirements applications. Through this method, policyholders receive the benefit of reduced required premium because they are credited with both investment income earned on Basic equity and with investment income earned on policyholder premiums associated with policies to be sold during PY 2014.

32. I could not replicate anything resembling Slide #8, when I went to Exhibits A.0.1, A.0.2, A.1.1 to A.1.4 for the derived discounted values, Exhibits F.1, F.2

Or

After finding on page 3-34 of the RRA

“D.9 CAPITAL MAINTENANCE

100. Exhibit Set G shows the calculation of capital maintenance, which is the cost of maintaining capital at the target level. Exhibit Set G also shows an allocation of Basic equity that is used to allocate investment income and capital maintenance to the coverage level. An increase in the amount of capital maintenance has an impact of +0.2 percentage points on the PY 2014 indicated rate change,” contrary to Slide #10. (underlining for effect)

<table>
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<tr>
<th>BASIC MAINTENANCE EXHIBIT G.2 (see lines)</th>
</tr>
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<tbody>
<tr>
<td>line (a)</td>
</tr>
<tr>
<td>145%</td>
</tr>
<tr>
<td>$ 1,179,589,000.00 x (1+4.73%)</td>
</tr>
</tbody>
</table>

33. Is it correct to say Figure 3.2 line item 8 Capital Maintenance of 0.2 % = $88.3 million and 145% MCT ? If not why not, and what is the value: $xx.x millions = 145% MCT ?

34. Although the following appears to fly contrary to page 3.35 of the RRA; quote:

“D.9.2 CAPITAL MAINTENANCE (EXHIBIT G.2)

104. In order to account for the growth in required capital over time, the Basic Capital Management Plan approved by the Commission calls for a provision to be included in the rate indication for capital maintenance. It specifies how the amount of capital maintenance should be calculated. For PY 2014, the amount of capital maintenance is $63.6 million. There are two elements used in determining the amount of capital maintenance, which have increased as compared to the 2013 Revenue Requirements Application: the transient target and the growth of required capital.

105. In the Decision on 2013 Revenue Requirements, the Commission approved a capital management target MCT ratio of 145%, and a 10 year transition period to bring the cost of maintaining capital at the target level into Basic insurance rates. The use of a transition period limits the impact on customers in a particular year, and is implemented by using a transient target MCT ratio for the determination of capital maintenance. The transient target MCT ratio, which was 110.5% for PY 2013, has increased to 114% for PY 2014 in line with the approved Basic Capital Management Plan.

106. The estimated growth of required capital has also increased compared to the 2013 Revenue Requirements Application. Referring to Figure 3.16, as a result of the higher rate of exposure growth compared with the 2013 Revenue Requirements Application, the
estimated growth in required capital is now slightly higher, so that the corresponding cost of maintaining an appropriate capital level has increased”.

35. Accordingly there is a conflict between my above exhibit “BASIC MAINTENANCE EXHIBIT G.2” which suggests a Capital Maintenance of $88.3 million verses paragraph 104 above of $63.6 million. (remember I am using ICBC’s formula given in Slide #8).

36. That said, is Figure 3.2 item 6 Capital Maintenance of 0.2 % = to $63.6 million and 145% MCT ? And how does ICBC spread this over the 10 year period ? Slide #13 really does not explain this. Could it be $63.6million divided by 10 years, if so, where is this accounted for in the RRA ?

37. After further research I came across ICBC IR#1 response to BCUC 2014.1 RR BCUC 1.1 Attachment A. Therein ICBC seems consistent with paragraph 104 above. So going in full circle, I believe ICBC mislead everyone, me at least with Slide #8 ?.  

38. Then Slide #10 adds a further katufel to the understanding of how Capital Maintenance is managed and calculated, while confirming the “Transient target of 114% MCT” with the Capital Maintenance of $63.6 million having an impact of 0.2% on the Indicated Rate Change.

39. Continuing from paragraph 30 herein and referring to Exhibit A.0.2 line (e) column 12 “Discounted Cash Flow”, ICBC refers to $56,621,000 as “Capital Maintenance Provision” in Exhibit A.0.1 line (n). This seems to conflict with the $63.6 million of Capital maintenance.

40. If the reader is now confused by this representation of ICBC 2014 RRA evidence so far, I have made my point.

41. In recap, presented so far; Capital Provision, Capital Maintenance, Discounted Cash Flow, Basic Equity are components that are used to allocate Investment Income and Capital Maintenance, Capital Build, and Transient Target of 114% MCT.

42. Adding to that is Slide #13; Slide #13 “Capital Build Formula” is misleading, go back to paragraph 36 herein.
43. From D.9.2 CAPITAL MAINTENANCE (EXHIBIT G.2) paragraph 105 page 3-35, we learn the “margin for adverse events” is not 100% to 130% but something in between. To repeat from para 105, quote:

“The transient target MCT ratio, which was 110.5% for PY 2013, has increased to 114% for PY 2014 in line with the approved Basic Capital Management Plan”.

44. The outstanding question is, what other benchmark ppts are there, and is transient target an adverse event? And who approved the transient increase from 110.5% to 114%, and was there any public / intervener process on this subject?

45. In Slide # 11 ICBC titles the slide “Capital Levels do not make rates higher for customers” The most misleading statement ICBC has made in this current application.

46. Since Capital Maintenance adds 0.2% to the Indicated Rate Change, then Capital directly impacts the Basic Insurance Rate and Premium paid!

47. It follows from the evidence hi-lighted from within the revenue requirement application Capital does affect rates.

48. Reminding the BCUC Commissioners that on September 18th ICBC issued their “Errata to ICBC's 2014 Revenue Requirements Application BIIS Sept 18, 2014”, to their Appendix 11B – Basic Insurance Information Sharing

49. In reply to ICBC’s errata filing on September 19th 2014, I submitted Intervener Exhibit C1-4. The following is my opening extract from the 4 page response; quote:

“ICBC found an error of 212 in every single spreadsheet/worksheet in the errata filing, no small error really. It demonstrates at least to this Intervener a common operator error known as “Global Data Entry”, wherein the operator selects a given “Cell” in the worksheet/spreadsheet for editing (Earned Premium Cell E12). The operator then selects “All” worksheets within the given spreadsheet file, and then types in the desired input to Cell E12 “212”, then hits the enter key, and every “cell” in each worksheet is updated with the 212 text, and then saves the file. Another danger noted in all the spreadsheets is the lack of interactive formula; ie: cell A1 1+2=3 ICBC changed the Earned Premium number from 212 with a different number for Cell E12 on each worksheet…. What’s going on here?”

50. In my response I question among other things why did ICBC actuaries sign off on these exhibits in the original 2014 RRA? What should be construed by the Commission and Interveners as to the reliability of Miss Camille Minogue, an employee of ICBC for some 10 years, and Mr. William Weiland, the Principal and Consulting Actuary with Eckler Ltd signatures. (see Appendix D for email copy). Where is the Independent Actuary Review?, meaning non ICBC employees. What assurances does ICBC offer that the resubmitted 105 page worksheets are accurate? Why did ICBC not submit along with this errata Miss Minogue re-certification?

51. Lastly, and I admit, what does this mean to the final outcome in the determination of the 5.2% Indicated Rate Change. Respectfully, I rely on the BCUC Chairman to confirm his satisfaction on this matter.
SENIOR CITIZEN CPP AND THE FEDERAL CPI

52. The two subjects CCP and CPI have been thoroughly discussed during this current 2014 RRA, so I am not going to burden the record with further discussion beyond these few words. I am submitting as evidence the most current CPP and CPI data, RTL Evidence #3-1 and #3-2 as evidence for consideration in regard to the limited resources British Columbian Senior citizens have to support themselves with, against the accepted actuarial practice manifestations of ICBC to increase the Basic Insurance Premiums far above Senior Citizens below inflationary (real dollar) pension incomes.

53. Special Direction IC2 requires ICBC to raise rates to cover costs, there is no point arguing this mandate. In the previous segment of this evidence I have discussed many aspects to question the reliability, modelling and accepted actuarial judgement ICBC has employed to justify their application for a 5.2% rate hike.

54. What should Senior Citizens give up in order to maintain their independent vehicle mobility? Why is the BCUC so against supporting Senior Citizens needs in preference to satisfy ICBC hungry money making machine?, based on highly questionable accepted actuarial practice, which has not been independently verified.

55. And before anybody mentions it, I do not qualify for the additional 25% disability allowance under the current British Columbian government guidelines.

56. So without the BCUC directing ICBC to adjust their Indicated Rate Change to mitigate the rate change for eligible Senior Citizens based on the Federal CPI, Senior Citizens will continue to remain disenfranchised without representation.

Discussion

57. In the BCUC decision for ICBC’s 2013 RRA, the Commission discussed updating evidence and the evidentiary usefulness of updated evidence. In summary the Commission is willing to accept evidence when clearly relevant to the application.

58. As mentioned in paragraph 52 herein RTL Evidence #3-1 and #3-2 are screen prints of the Federal Governments latest update information as they pertain to the CPP and the CPI.

59. The Federal Government has calculated that the CPP payments to Senior citizens across the country will not be getting a pension increase in 2015 as the CPI indicated the overall annual CPI dropped -1.8% for 2014. This conclusion means a further disparity between British Columbian Senior Citizens pension income to ICBC Basic Insurance Premium rate increase of 5.2% for 2014 RRA

60. On the evidence ICBC Basic Insurance Premium rate went up 5.2% for the 2013 RRA, where in the BCUC approved the 4.9% rate increase as requested by ICBC, and added a further 0.3% for some ICBC employee Pension calculation adjustments, becoming effective November 1st 2014

61. If the BCUC approve the current 2014 RRA of 5.2% to cover costs as submitted by ICBC, then BC Senior Citizens will now incur an accumulated 10.4% spread between their CPP income pegged in January 2014 and ICBC’s horrific rate increases over the last two year period. This follows on the heels of the 11.2% award approved by the BCUC in ICBC’s 2011/12 RRA.

To amass a grand total of 21.6% since 2011.
62. By any stretch of the imagination, this compounding effect cannot be translated into saying ICBC costs have jumped 21.6%, or that the precious *Special Direction IC2* for rate smoothing and predictability ± 1.5% is justifiable.

63. The BCUC must employ a true “Independent Actuarial Review” before reaching a decision for this current 2014 RRA.

64. Effectively since 2011 the Canadian Pensions income to British Columbian Senior Citizens has been indexed up only by 5.3% versus ICBC Basic Insurance Premium rates of 21.6%,

- four times! the CPP rates
65. I have submitted my RTL Evidence 2.1, 2.2 and 2.3 (data sheet) before in 2013, it has been updated to reflect the record and this current 2014 application.

66.1 ICBC actuaries signed off on a 105 page 11B exhibit that were wrong. ICBC offered no explanation for the sign off error, or the actual errors. ICBC offered no information in terms of in-house corrective measures to avoid a recurrence of such spreadsheet management errors in the future.

66.1.2 Appendix D - Actuary Certification is a look back in history to 2006, 2007 and 2014 of Actuary Certifications given in each RRA for each year. I am sure there are others ??? The wording in each certificate for each year is “identical”, and according to Miss Minogue she has reviewed in, quote: “All rate class categories” and Miss Minogue goes onto say “I believe the data is reliable and sufficient for the determination of the indicated changes in average rate level”. An extensive review by the BCUC of ICBC record filings will show a huge listing of errata filings. Which in my opinion the BCUC should investigate, with the view of providing better ICBC Actuary review instruction, upgrading review processes and certifications. This hi-light is not interested in “accepted practice”, but rather the historical filing record of accurate information for the BCUC and the independent determination of data to support any indicated rate change, (in past, present and future filings).

66.2 ICBC remains mute in regard to this Intervener’s C1-4 response submission filed September 19th 2014 to the ICBC errata corrective filing September 18th 2014 regarding the Chapter 11B exhibit.

66.3 ICBC misrepresented material information during the Review Workshop to the BCUC and Interveners attending with various slides that refer to such things and more; Capital Maintenance, Capital Build, MCT formula calculations, and how the MCT does not add to the Indicated Rate Change.

66.4 ICBC exhibits within the 2014 RRA do not provide adequate analysis and discussion to disseminate or support the independent analysis and independent conclusions by others.

66.5 ICBC increases Basic Premium Insurance rates by 21.6% over the last three Revenue Requirement Applications – 2011/12, 2013 and 2014, under the protection of “Covering Costs” and Special Direction IC2. Without any intervention by the BCUC, but rather creative support, ie: 0.3% in their 2013 decision.

66.6 Intervener evidence submitted herein identifies by way of simplification while using “Bodily Injury – Personal” as an example, how actuarial judgement and preferential selection can predetermine a desired outcome. While providing analysis and discussion to support the judgement and selection criteria, albeit an over simplification, as is the email spam example.

66.7 Intervener evidence submitted herein demonstrates significant disparity between ICBC Basic Insurance Premium compounding rate increases since 2011 – 21.6% versus the Federal Government CPP and CPI increases of 5.3% over the same period.

66.8 Intervener evidence submitted herein, namely: “RTL–Evidence 2.1, 2.2, 2.3” demonstrates graphically these assertions described above in paragraph 66.7 It is noted ICBC may dispute the graphic depictions as they have in the past. But with the updates to “RTL Evidence 2.3” as highlighted in yellow, over time (succeeding RRA’s), this evidence will prove itself, especially in the future record following the implementation of Special Direction IC 2 for Rate Smoothing and Rate Predictability as will be demonstrated in the succeeding years.... !
66.9.1 Appendix B - Bornhuetter-Ferguson Principle is provided to the Commission because ICBC uses the actuarial principles described in this paper (as per Exhibit C.1.3.1 – Basic Bodily Injury Incurred Loss Summary describes). I make no bones about it, I am totally befuddled by it, while I respect basic principles described in this Appendix B paper as a reliable actuarial model.

66.9.2 Although, I am requesting the BCUC to thoroughly question ICBC actuarial judgements, decision making criteria, data selection sets and weighting factor selections in an effort to have ICBC justify on record their selections and arithmetic. Since ICBC has not provided their analytic choices / selections, criteria, principles and decisions as ICBC apparently claims to follow Special Direction IC2 without detailed descriptive explanations.

66.9.3 Try following two selected Weighting Factors and their impact upstream / downstream in Exhibit C.1.2.3, namely “Selected CDFs 1.0138” and “Aged-to-Ultimate Factors 1.0241” using excel spreadsheet formula, it can not be done!

66.9.4 The reasons for my questioning the Bornhuetter-Ferguson Principle methods, along with all the other “methods” (see Appendix C for a summary listing derived from the noted exhibits), is because ICBC fails to provide descriptive analysis and discussion of their choices; (again I cite the words given in Exhibit C.1.0 para 1., (quote: “revenue requirements analysis is discussed below”). Without descriptive discussion / text, how can anyone reach their own independent conclusions, whether in agreement or in objection. Wherein the BCUC and/or any Intervener acting independently of ICBC can argue on the same equal footing, while having parity, and equality in their subsequent evidentiary filings.

66.10 From Section 3 the following are named ICBC models, also compare with Appendix C:

### A PLATE OWNER BASIC BODILY INJURY (EXHIBITS C.1.0 TO C.1.4.9 AND C.5.0 TO C.5.4.9)

3. A summary of the exhibits underlying the selected incurred loss and ALAE amounts is shown below.

<table>
<thead>
<tr>
<th>Description</th>
<th>Personal Exhibits</th>
<th>Commercial Exhibits</th>
</tr>
</thead>
<tbody>
<tr>
<td>Summary of Selections</td>
<td>C.1.0 to C.1.1.1</td>
<td>C.5.0 to C.5.1.1</td>
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<td>Counts</td>
<td>C.1.2.1 to C.1.2.3</td>
<td>C.5.2.1 to C.5.2.3</td>
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<td>Incurred Loss Summary</td>
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<td>C.5.3.2 to C.5.3.6</td>
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<td>Paid Development Method</td>
<td>C.1.3.7 to C.1.3.17</td>
<td>C.5.3.7 to C.5.3.17</td>
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<tr>
<td>Bornhuetter-Ferguson Method</td>
<td>C.1.3.18 to C.1.3.19</td>
<td>C.5.3.18 to C.5.3.19</td>
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<td>Incurred ALAE Summary</td>
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<td>Paid ALAE Development Method</td>
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<td>C.5.4.2 to C.5.4.5</td>
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<td>Incremental Paid ALAE to Incurred Loss Method</td>
<td>C.1.4.6 to C.1.4.8</td>
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<tr>
<td>Bornhuetter-Ferguson Method (ALAE)</td>
<td>C.1.4.9</td>
<td>C.5.4.9</td>
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</table>

There are many more....!

What do “YOU” know about them?
EVIDENTARY CONCLUSIONS

67 Based on the evidenced offered herein for the BCUC Commissioners to examine, I assert the limitations of my evidence is based on the lack of analysis and discussion by ICBC in their 2014 RRA filing exhibit B-3 and all succeeding exhibit filings to date.

68 It is impossible to me to provide the Commissioners with an alternative recommended rate increase based on the assertions described above. And it should not be construed from the lack of recommendation as in agreement with ICBC’s Indicated Rate Change.

further;

69 ICBC filed with their 2014 RRA “pdf format” excel spreadsheets, and although I cannot remember how, I have obtained all the 2014 RRA Exhibit excel spreadsheets, (I must have asked for them). Except they are totally useless, as ICBC turned off all the Check formula, Evaluate formula, Trace precedents and Trace dependants features, rendering Error checking also useless.

70 By turning off the Error Checking feature, the errata filing by ICBC on September 18th became necessary, regardless of the Actuarial Certification filed with the application.

71 ICBC denied me the ability to present my evidence on an equal footing. I am unable to review their analysis and discussion in such a manner as to independently apply any, some or all my own conclusions in review of the excel spreadsheet exhibits submitted by ICBC in this 2014 RRA.

72 At best my evidence can only pick holes in ICBC’s 2014 RRA.

73 As in the past both the BCUC Commissioners (in their decisions) and ICBC have dismissed any responsibility for their actions in regard to British Columbian Senior Citizens limited Canada Pension incomes to coup, or at least stay abreast of ICBC Basic Insurance Premium increases.

74 My Intervener evidence over time will prove themselves, as has been demonstrated between 2013 and 2014 figures show. The following takes a leaf from ICBC’s 2013 Annual Report page 57, (see Appendix C), quote:

“based on historical experience”

75 Since becoming a registered intervener in 2011 in every ICBC RRA since, I have learned that not one Intervener has ever been able to assert a viable opinion to challenge ICBC “Indicated Rate Change”,

why is that?

76 To repeat myself, British Columbian Senior Citizens are unable to keep pace with their CPP to ICBC Basic Insurance Premiums rate increases over the last three RRA’s

CPP 5.3% versus ICBC 21.6%
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<th>2014</th>
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<th>2013</th>
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<td>385,730,400.00</td>
<td>500,396,160.00</td>
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<td>% Total Capital</td>
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<td>100%</td>
<td>100%</td>
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<td>23.8%</td>
<td>72.0%</td>
<td>83.5%</td>
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<tr>
<td>Accum MCT Total</td>
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<td>145.5%</td>
<td>153.8%</td>
<td>172.0%</td>
<td>183.5%</td>
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<td>-</td>
<td>-</td>
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<tr>
<td>MCT Level</td>
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<td>100%</td>
<td>100%</td>
<td>130%</td>
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<tr>
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<td>-</td>
<td>-</td>
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<td>160%</td>
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<tr>
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<td>-</td>
<td>-</td>
<td>-</td>
<td>12.0%</td>
<td>23.5%</td>
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**Solvency MCT**
- 100%
- 130%

**Excess MCT Solvency**
- 1.0%
- 15.5%
- 23.8%
- 72.0%
- 83.5%

**Accum MCT Total**
- 131.0%
- 145.5%
- 153.8%
- 172.0%
- 183.5%

**Rate Smoothing MCT Level**
- 145%
- 145%

**CRC Ceiling**
- 160%
- 160%

**CRC Refund MCT**
- 12.0%
- 23.5%
# Actuarial Weighted MCT Calculator Based on Bodily Injury Weighting Value $200,000,000 (50% of Base)

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<th>Weighting Factor</th>
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<th>BI Reqd</th>
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<tr>
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<tr>
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<th>% Total Capital Incr'd</th>
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<td>2015</td>
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<td>2015</td>
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# ACTUARIAL WEIGHTED MCT CALCULATOR BASED ON BODILY INJURY WEIGHTING VALUE $200,000,000 (25% of BASE)

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<th>2015</th>
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<tr>
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<td>0.25</td>
<td>0.3</td>
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<tr>
<td>Capital Reqd</td>
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<td>154,482,768.00</td>
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<td>9.0%</td>
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<tr>
<td>Average</td>
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<td>Average</td>
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Solvency MCT

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Excess MCT Solvency

<table>
<thead>
<tr>
<th></th>
<th>1.0%</th>
<th>15.5%</th>
<th>23.8%</th>
<th>72.0%</th>
<th>83.5%</th>
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Accum MCT Total

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<tr>
<th></th>
<th>131.0%</th>
<th>145.5%</th>
<th>153.8%</th>
<th>172.0%</th>
<th>183.5%</th>
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<table>
<thead>
<tr>
<th></th>
<th>Rate Smoothing</th>
<th>MCT Level</th>
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<tbody>
<tr>
<td></td>
<td>-</td>
<td>145%</td>
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<table>
<thead>
<tr>
<th></th>
<th>CRC Ceiling</th>
<th>CRC Refund MCT</th>
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<tr>
<td></td>
<td>-</td>
<td>12.0%</td>
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RTL EVIDENCE #1-4
<table>
<thead>
<tr>
<th>Line</th>
<th>Components in Figure 3.2</th>
<th>Rate Impact for PY 2014 (£)</th>
<th>Components in Exhibit A, p. 7</th>
<th>PY 2014 (£)</th>
<th>PY 2013 (Adjusted for Policy Growth) (£)</th>
<th>Change From Previous PY (E) (£)</th>
<th>% Increase From Current Rate Level (%)</th>
<th>Projected Premium at Current Rate Level (£)</th>
<th>Percentage Point Indicated Rate Change (%)</th>
<th>Reference for (d)</th>
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<tbody>
<tr>
<td>1</td>
<td>Discontinuing P/O Year’s Rate Exclusion$^1</td>
<td>0.4%</td>
<td>n/a</td>
<td>£ 230,117.01</td>
<td>230,117.01</td>
<td>0.4%</td>
<td>230,117.01</td>
<td>230,117.01</td>
<td>0.4%</td>
<td>No rate exclusion in FY 2014 – Chapter 3, Figure 3.2, Line (11)</td>
</tr>
<tr>
<td>2</td>
<td>Loss and A&amp;AE Payments$^2</td>
<td>1.6%</td>
<td>127,903.00</td>
<td>239,117.01</td>
<td>111,214.00</td>
<td>0.4%</td>
<td>239,117.01</td>
<td>239,117.01</td>
<td>0.4%</td>
<td>Exhibit 4.8, Line (a)</td>
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<tr>
<td>3</td>
<td>PY 2013 Loss Cost Fluctuation Variance</td>
<td>-1.5%</td>
<td>127,903.00</td>
<td>239,117.01</td>
<td>-111,214.00</td>
<td>-0.4%</td>
<td>239,117.01</td>
<td>239,117.01</td>
<td>-0.4%</td>
<td>Please see the response to information request 2014.1 RR BCUC 4.1-6, Attachment 4, Row (1), Col (h), multiplied by policy year 2014 TDI Basic exposures</td>
</tr>
<tr>
<td>4</td>
<td>Loss Trend to PY 2014</td>
<td>2.9%</td>
<td>127,903.00</td>
<td>239,117.01</td>
<td>111,214.00</td>
<td>0.4%</td>
<td>239,117.01</td>
<td>239,117.01</td>
<td>0.4%</td>
<td>Exhibit 4.9, Line (a)</td>
</tr>
<tr>
<td>5</td>
<td>Investment Income</td>
<td>-0.1%</td>
<td>127,903.00</td>
<td>239,117.01</td>
<td>-111,214.00</td>
<td>-0.4%</td>
<td>239,117.01</td>
<td>239,117.01</td>
<td>-0.4%</td>
<td>Exhibit 4.9, Line (a)</td>
</tr>
<tr>
<td>6</td>
<td>Operating Expense</td>
<td>-0.1%</td>
<td>127,903.00</td>
<td>239,117.01</td>
<td>-111,214.00</td>
<td>-0.4%</td>
<td>239,117.01</td>
<td>239,117.01</td>
<td>-0.4%</td>
<td>Exhibit 4.9, Line (a)</td>
</tr>
<tr>
<td>7</td>
<td>Capital Maintenance</td>
<td>0.2%</td>
<td>127,903.00</td>
<td>239,117.01</td>
<td>111,214.00</td>
<td>0.4%</td>
<td>239,117.01</td>
<td>239,117.01</td>
<td>0.4%</td>
<td>Exhibit 4.9, Line (a)</td>
</tr>
<tr>
<td>8</td>
<td>Other</td>
<td>0.1%</td>
<td>127,903.00</td>
<td>239,117.01</td>
<td>111,214.00</td>
<td>0.4%</td>
<td>239,117.01</td>
<td>239,117.01</td>
<td>0.4%</td>
<td>Exhibit 4.9, Line (a)</td>
</tr>
<tr>
<td>9</td>
<td>Other components of Required Premium at A/0.1</td>
<td>0.1%</td>
<td>127,903.00</td>
<td>239,117.01</td>
<td>111,214.00</td>
<td>0.4%</td>
<td>239,117.01</td>
<td>239,117.01</td>
<td>0.4%</td>
<td>Exhibit 4.9, Line (a)</td>
</tr>
<tr>
<td>10</td>
<td>Difference in Value of 6/14 Risk Exclusions</td>
<td>0.2%</td>
<td>127,903.00</td>
<td>239,117.01</td>
<td>111,214.00</td>
<td>0.4%</td>
<td>239,117.01</td>
<td>239,117.01</td>
<td>0.4%</td>
<td>Exhibit 4.9, Line (a)</td>
</tr>
</tbody>
</table>

This table is copied from ICBC IR#1 Response to BCUC 2014.1 RR BCUC 1.1 Attachment A
A 1.5% Rate Smoothing adder will increase ICBC’s Revenue over the next 10 Premium Year cycle by $396. Multiply this number by 3.4 million other Basic Premium Policy Holders and ICBC receives a whopping $1,349,247,645.82 which is $293,311,907.20 projected premium revenue increase over the 17 year period.

**BASIC PREMIUM INCREASES WITH 1.5% RATE SMOOTHING BETWEEN 2009 AND 2025**

**EFFECTIVE COMPOUNDED ANNUAL GROWTH RATE OF BASIC PREMIUM INSURANCE BETWEEN 2014 TO 2025**

**BASIC PREMIUM RATE INCREASES BETWEEN 2009 AND 2025, BUT BEFORE CPI**

**BASIC PREMIUM + 1.5% RATE SMOOTHING + 1.2% ANNUAL INFLATION BETWEEN 2014 TO 2025**

ICBC Compounding Rate Increases.xls

RTL-EVIDENCE 2.1

BPR=BASIC PREMIUM RATE, CPI=CANADA PENSION INFLATION
ICBC 2014 REVENUE REQUIREMENT APPLICATION
BASIC PREMIUM IMPACT GRAPHS

Prepared by: Ricahrd T. Landale

BPR = BASIC PREMIUM RATE

Over the next 10 Premium Years, when both a modest 1.2% CPI (2009 to 2014 incl CPI updated) plus 1.5% are compounded together (CAGR), ICBC receives another whooping injection of $21,696.60 Revenue per policy, because of the 1.5% Rate Smoothing effect. Multiply that by 3.4 million other Policy Holders, and ICBC receives $73,768,449,736.70 (up from $54,233,903,944.44)

Accumulative Basic Premium Income $21,696.60 up from $15,951.15
**Definition of 'Compound Annual Growth Rate - CAGR'**

The year-over-year growth rate of an investment over a specified period of time.

**BPR = BASIC PREMIUM RATE, CPI - CANADA PENSION INFLATION RATE**

<table>
<thead>
<tr>
<th>CAGR GRAPH DATA TABLES</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>YEAR</strong></td>
</tr>
<tr>
<td>----------------</td>
</tr>
<tr>
<td>1 2009</td>
</tr>
<tr>
<td>2 2010</td>
</tr>
<tr>
<td>3 2011</td>
</tr>
<tr>
<td>4 2012</td>
</tr>
<tr>
<td>5 2013</td>
</tr>
<tr>
<td>6 2014</td>
</tr>
<tr>
<td>7 2015</td>
</tr>
<tr>
<td>8 2016</td>
</tr>
<tr>
<td>9 2017</td>
</tr>
<tr>
<td>10 2018</td>
</tr>
<tr>
<td>11 2019</td>
</tr>
<tr>
<td>12 2020</td>
</tr>
<tr>
<td>13 2021</td>
</tr>
<tr>
<td>14 2022</td>
</tr>
<tr>
<td>15 2023</td>
</tr>
<tr>
<td>16 2024</td>
</tr>
<tr>
<td>17 2025</td>
</tr>
</tbody>
</table>

**Increases $ 1,255.84 35.70% Effective Compound Increase over term 156.98% Annualized Compound Rate over 17 years 5.71%**

<table>
<thead>
<tr>
<th><strong>BPR + 1.2% Inflation</strong></th>
<th><strong>Compounded BPR + Inflation</strong></th>
<th><strong>Compounded Revenue Rate</strong></th>
<th><strong>Compounded Premium Income</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>1 2009</td>
<td>$ 800.00 0.00%</td>
<td>$ 800.00</td>
<td>0.00%</td>
</tr>
<tr>
<td>2 2010</td>
<td>$ 1,275.33 6.25%</td>
<td>$ 1,338.00</td>
<td>6.25%</td>
</tr>
<tr>
<td>3 2011</td>
<td>$ 1,418.17 0.00%</td>
<td>$ 1,306.00</td>
<td>0.00%</td>
</tr>
<tr>
<td>4 2012</td>
<td>$ 1,577.01 11.20%</td>
<td>$ 1,577.00</td>
<td>11.20%</td>
</tr>
<tr>
<td>5 2013</td>
<td>$ 1,659.01 5.20%</td>
<td>$ 1,838.86</td>
<td>16.60%</td>
</tr>
<tr>
<td>6 2014</td>
<td>$ 1,745.27 5.20%</td>
<td>$ 2,061.06</td>
<td>12.08%</td>
</tr>
<tr>
<td>7 2015</td>
<td>$ 1,766.22 1.20%</td>
<td>$ 2,333.67</td>
<td>13.23%</td>
</tr>
<tr>
<td>8 2016</td>
<td>$ 1,787.42 1.20%</td>
<td>$ 2,610.27</td>
<td>11.85%</td>
</tr>
<tr>
<td>9 2017</td>
<td>$ 1,808.86 1.20%</td>
<td>$ 2,892.09</td>
<td>10.80%</td>
</tr>
<tr>
<td>10 2018</td>
<td>$ 1,830.57 1.20%</td>
<td>$ 3,180.11</td>
<td>9.96%</td>
</tr>
<tr>
<td>11 2019</td>
<td>$ 1,852.54 1.20%</td>
<td>$ 3,475.15</td>
<td>9.28%</td>
</tr>
<tr>
<td>12 2020</td>
<td>$ 1,874.77 1.20%</td>
<td>$ 3,777.97</td>
<td>8.71%</td>
</tr>
<tr>
<td>13 2021</td>
<td>$ 1,897.27 1.20%</td>
<td>$ 4,089.23</td>
<td>8.24%</td>
</tr>
<tr>
<td>14 2022</td>
<td>$ 1,920.03 1.20%</td>
<td>$ 4,399.55</td>
<td>7.83%</td>
</tr>
<tr>
<td>15 2023</td>
<td>$ 1,943.07 1.20%</td>
<td>$ 4,739.52</td>
<td>7.48%</td>
</tr>
<tr>
<td>16 2024</td>
<td>$ 1,966.39 1.20%</td>
<td>$ 5,079.71</td>
<td>7.18%</td>
</tr>
<tr>
<td>17 2025</td>
<td>$ 1,989.99 1.20%</td>
<td>$ 5,430.67</td>
<td>6.91%</td>
</tr>
</tbody>
</table>

**Accumulative Totals $ 21,696.60 $ 1,055,935,738.62**

Notes to reader:
- This Data Sheet is the source data to generate RTL- Evidence 2.1 and 2.2
- I apologize for not providing the formulas, I do not know how to include them.
- Data sheet formatting is also for personal convenience.
- Data hi-lighted in yellow has been updated from 2013 RRA

ICBC Compounding Rate Increases.xls DATA

RTL - EVIDENCE 2-3
Federal Government calculations for the determination of the Consumer Price Index and the resultant index for the Federal Canada Pension Plan for 2015 - CPI is -1.8% for the 2014 year.
Federal Government Consumer Price Index as of December 19th 2014

Please note the Federal Government has indicated the CPI did increase 2.0 ppts November 2013 to November 2014. And while the overall CPI for 2014 reflects a -1.8%, the overall CPI did go up, further increasing the disparity.

(Website screen capture showing Consumer Price Index table and statistics.)

RTL EVIDENCE #3.2
Ms. Erica Hamilton  
Commission Secretary  
British Columbia Utilities Commission  
Sixth Floor,  
900 Howe Street  
Vancouver,  
British Columbia  
V6Z 2N3

Dear Ms Hamilton,

Re: Insurance Corporation of British Columbia  
2014 Revenue Requirements Application  
Intervener C1 - Response to ICBC Errata submission September 18th 2014

Please find the following in response to ICBC errata submission on September 18th 2014.

As this is a streamlined process, and no clear process to follow, I am making this submission immediately, as I believe the matters discussed in this submission to be very important to both the BCUC and Commissioners, and to ICBC.

This timely response also affords ICBC time to address the issues discussed ahead of the Review Working Session on September 26th 2014

Kind regards,

[Signature]

Richard T. Landale
cc: ICBC and Registered Interveners
Intervener C1 - Response to ICBC Errata submission September 18th 2014

On September 18th ICBC issued their “Errata to ICBC’s 2014 Revenue Requirements Application BIIS Sept 18, 2014.pdf”

ICBC found an error of 212 in every single spreadsheet/worksheet in the errata filing, no small error really. It demonstrates at least to this Intervener a common operator error known as “Global Data Entry”, wherein the operator selects a given “Cell” in the worksheet/spreadsheet for editing (Earned Premium Cell E12). The operator then selects “All” worksheets within the given spreadsheet file, and then types in the desired input to Cell E12 “212”, then hits the enter key, and every “cell” in each worksheet is updated with the 212 text, and then saves the file. Another danger noted in all the spreadsheets is the lack of interactive formula; ie: cell A1 1+2=3 ICBC changed the Earned Premium number from 212 with a different number for Cell E12 on each worksheet.... What’s going on here?

Another important error to note for just one example: “Worksheet 2BI – 1” cells E11-to E15 using the 2010 year at (cell E12=212), if the Actuaries had done due diligence, they would have noticed Cell E16 Total was in error, at 6,316,261,929, but it was reported as 7,840,260,136. This typo error was repeated on every worksheet in the file. Another question begs, why did ICBC turn off the automatic update feature in the entire Spreadsheet file to avoid running Totalization errors? The magnitude of this error can only be considered as catastrophic. And thereby impugn the integrity of all other spreadsheets used by ICBC Actuaries in the composition of this PY 2014 RRA.

Based on my review I suggest worksheet 2BV-56 Cell 12 could well be the clue of how this 212 error occurred?

The issue to be contemplated by the BCUC and the Commissioners is the reliability of ICBC’s Excel spreadsheet.xlsx files. It is obvious there is an accuracy and integrity and quality assurance problem.

Is the problem an isolated event in just this PY 2014 RRA, or is there a history of errors as in PY 2013 RRA and before? Obviously not, especially for 2009 and 2010 years. I also ask the BCUC to reflect on the various corrections made during the PY2013 RRA process and in the Oral Hearings (various handouts).

To amplify my concern, I remind the BCUC and Commissioners of my address during the IR’s and in the Oral Hearing in regard to the Certified Actuary and the Review Actuary, PY2014 RRA pages 3-39 and 3-40 respectively. The errors in PY2014 are errors these people signed off on. Their signatures are meant to provide integrity, quality assurance that “Accept Actuarial Practice” has been properly applied within the application. Which in turn is intended to give the BCUC and Commissioners the understanding the Actuaries under took due diligence in their preparations and review.

Nobody signs off on errors if they “actuarially” looked at the spreadsheets, do they?

Is an actuary a person who: is a statistician who calculates insurance premiums, risks, dividends, and annuity rates?..... I won’t finish the sentence.

It is true, I have found selective passages to make a point, the point, ICBC claims in different words the essence of professional due diligence. (I have underlined certain parts for effect)

Consider the words of Mr. Ghikas in Volume 6 of the PY2013 Oral Hearings page 883,884
MR. GHIKAS: Mr. Chairman, if I can just add, prospective adjustments, there is a section in the application ICBC - Revenue Requirements that deals specifically with prospective adjustments.

It’s actually in the actuarial exhibits, and it’s the exhibit set is -- so it’s Chapter 3, exhibit sets E-1 to E-6, and the rules around when you can and cannot include prospective adjustments are actually dictated by actuarial practice.
Question, in PY2014 Chapter 11 Exhibits, are they under the same rules as Chapter 3, so mentioned by Mr. Ghikas?

It seems illogical that over 100 (105) pages of Actuarial exhibits in Chapter 11 having inherent errors a “prospective adjustment”? The only answer to that can come from a truly “Independent Review” by companies like “Price Waterhouse” for example. *(I have no affiliation)*

Later on in the Oral Hearing on pages 979, 980 Ms Prior responds:

MS. PRIOR: A: No, I think -- well, okay, let me clarify. *When we were thinking about the streamlined process, what we were saying is, because there is actually a band defined in regulation of plus or minus 1.5 percentage points around whatever the Commission decides at this hearing, that we would suggest a streamlined process. And that streamlined process would be similar to the past streamlined processes that we used before in 2010, which is we would go through a process, we would put our filing together Some areas we would suggest could be a little narrower in scope than what we actually submit, but the full actuarial information would be there. And it would have a round of IRs. It wouldn’t be an oral hearing process, if that’s what you’re getting at, Mr. Munn, but it would be basically the documents would be submitted. We would do one round of IRs. This is similar to what we did in 2010.*

MR. MUNN: Q: *Now, what if -- just a couple of “what ifs”. What if we’re looking at an actual increase of 6.4 in 2014, and then the further 1.5, so 7.9 in 2015. Is there a point where the size of the rate increase should require that we come back for a more fulsome hearing with the Commission?*

MS. PRIOR: A: *Well, I think there you’d have to look at a number of factors to determine, you know, how, number 1, the actual rate amount. I think if we had three subsequent years and you were up to the 7.9 or into the 8s, I think that would be concerning for all of us. And we would also be looking at where our capital levels are. What this -- what the actual framework allows is for ICBC to come back at any point in time and do an additional annual filing. That may be not a streamlined process, that may be a full oral hearing or whatever other process the Commission determines.*

Wow Ms. Prior, you called that right…!

Ms Prior continues discussion on page 982:

MS. PRIOR: A: *Well, I think we should take it one step at a time. I think too, number one, this rate smoothing is new to all of us. And we need to see how that evolves. I think in the -- we’re looking only at the next hearing, which is -- or the next process, which is coming up very quickly. And what we’re suggesting is going with the streamlined application for that process.*

Again you called it…..!

Ms Prior continues discussion on page 1066:

MS. PRIOR: A: *I think if you are talking about what numbers go into the revenue requirement, I think you actually have to go to the actuarial section of the filing, because they do things on a policy year basis, not a calendar year basis.*

If that is true, what is Chapter 11 all about ?, and why did ICBC “NEED” to file the Errata on September 18th, 2014 ? Why set History straight…. 2009 and 2010 ?

The Chairman on pages 1140,1141 makes the following address:

THE CHAIRPERSON: And let me just add to that. I will echo what Commissioner O’Hara has said about following the actuarial guidelines, and making sure that we stick to that. But at the same time, we want to be aware of the sensitivities around that because of the very real decision that we have to make on rates going forward. And it is one where we’re basically making a three-year decision. Panels that come after us will have
some leeway, but they’re restricted by IC-2 as to what the rate can be granted. And we’re also aware of the changes that we might make on the MCT targets, in terms of the 150 and the requested 165 and 173. So, that’s all we’re asking for is so we have sufficient information to provide us with the knowledge to be able to make the best decision possible within the boundaries that we have available to us.

If I understand the Chairman’s comments, the Commissioner relies on accurate information, reliable information, and consistent information year to year, so that their current decisions are appropriate in future years. One begs the following questions for PY2014 RRA, how accurate is the actuarial spreadsheet data exhibits, as this data manifests itself through to the determination of the 5.2PPT Indicated Rate Change, when Premium Earned data is skewed, and requires corrections at a later date? Incidentally I hope the BCUC and Commissioners question... why now? why updating 2009 and 2010 data? and does this skewed data have impacts on previous Rate applications in 2011, 2012, 2013 and 2014? Also consider how Actuaries rely on historical data for trends and trend analysis? (we’ve heard that a few times).

Lastly.... What has changed in PY2014? I refer to ICBC reply March 17th 2014 - page 30 para 57: 57. Mr. Landale is also critical of ICBC’s actuaries. Mr. Landale says, for instance, “Based on the outrageous “Accepted Actuarial Practice” code, ICBC actuaries have developed a hypothesis of “Scare Tactic” by being biased and exaggeratory (sic) in the Loss Cost Forecast Variance...[etc. etc.]”. ICBC’s actuaries are professionals and there is every reason to believe that they will conduct themselves as such. They have indicated that the selection of models and assumptions is an exercise of informed judgment. By selecting modelling that accords with accepted actuarial practice and best accounts for the available information, ICBC’s actuaries are doing exactly what they are mandated to do both by their professional standards of conduct and by Special Direction IC2.

Hmmm.!
The Bornhuetter-Ferguson Principle

by Klaus D. Schmidt and Mathias Zocher

ABSTRACT

The present paper provides a unifying survey of some of the most important methods of loss reserving based on run-off triangles and proposes the use of a family of such methods instead of a single one.

The starting point is the thesis that the use of run-off triangles in loss reserving can be justified only under the assumption that the development of the losses of every accident year follows a development pattern which is common to all accident years.

The notion of a development pattern turns out to be a unifying force in the comparison of various methods of loss reserving, including the chain-ladder method, the loss-development method, the Cape Cod method, and the additive method. For each of these methods, the predictors of the ultimate losses can be given the shape of Bornhuetter-Ferguson predictors.

The process of arranging known methods of loss reserving under the umbrella of the extended Bornhuetter-Ferguson method requires the identification of prior estimators of the development pattern and the expected ultimate losses. This process can be reversed by combining components of different methods to obtain new versions of the extended Bornhuetter-Ferguson method.

The Bornhuetter-Ferguson principle proposes the simultaneous use of various versions of the extended Bornhuetter-Ferguson method and a comparison of the resulting predictors in order to select best predictors and to determine prediction ranges.

KEYWORDS

Additive method, Bornhuetter-Ferguson method, Bornhuetter-Ferguson principle, Cape Cod method, chain-ladder method, development pattern, loss-development method, loss reserving, run-off triangle
1. Introduction

During the last decades, actuaries have proposed a large variety of methods of loss reserving based on run-off triangles. In each of these methods, it is assumed that all claims are settled within a fixed number of development years and that the development of incremental or cumulative losses from the same number of accident years is known up to the present calendar year such that the losses can be represented in a run-off triangle.

The most venerable and most famous of these methods are certainly the chain-ladder method and the Bornhuetter-Ferguson method. It appears that the basic idea of the chain-ladder method was already known to Tarbell (1934) while the Bornhuetter-Ferguson method was first described almost forty years later in the paper by Bornhuetter and Ferguson (1972).

At the first glance, both methods have very little in common:

- The chain-ladder method proposes predictors of the ultimate (cumulative) losses and every predictor is obtained sequentially by multiplying the current (cumulative) loss by the chain-ladder factors which are certain development factors (or link ratios) obtained from the run-off triangle.
- The Bornhuetter-Ferguson method proposes predictors of the outstanding losses and every predictor is obtained by multiplying an estimator of the expected ultimate (cumulative) loss by an estimator of the percentage of the outstanding loss with respect to the ultimate one.

The fact that these methods aim at different target quantities can be neglected since predictors of ultimate losses can be converted into predictors of outstanding losses, and vice versa. However, a crucial difference lies in the fact that the chain-ladder method proceeds from current losses while the Bornhuetter-Ferguson method is based on the expected ultimate losses, and this difference is connected with the sources of information which are taken into account:

- The chain-ladder method relies completely on the data contained in the run-off triangle.
- The Bornhuetter-Ferguson method restricts the use of the run-off triangle to the estimation of the percentage of the outstanding loss and uses the product of the earned premium and an expected loss ratio to estimate the expected ultimate loss.

The aim of this paper is to show that, in spite of their different appearances, the chain-ladder method and the Bornhuetter-Ferguson method, as well as many other methods of loss reserving, have indeed very much in common.

The striking point with the Bornhuetter-Ferguson method is the multiplicative structure of the predictors of the outstanding losses. In the original version of the method, each of the two factors has the particular meaning mentioned before. This interpretation may be dropped. If it is dropped, then we obtain predictors each of which is the product of some estimator of the ultimate loss and some estimator of the percentage of outstanding losses, and we are free to choose these estimators as we like.

We thus arrive at a general class of predictors of outstanding losses, and hence at a corresponding class of predictors of ultimate losses.1 The predictors of this class will be referred to as predictors of the extended Bornhuetter-Ferguson method. It will be shown that the chain-ladder predictors belong to this class and that the predictors of many other methods of loss reserving belong to this class as well.

Since the prediction of outstanding or ultimate losses is a statistical problem, it is most helpful to formulate all methods (which in many cases

1Like the chain-ladder method and the Bornhuetter-Ferguson method, different methods of loss reserving address different target quantities. For the sake of comparison, it is necessary to express all methods in terms of the same target quantities. Our choice of focusing on ultimate losses (and later also on other cumulative losses) is essentially a matter of personal preferences.
The Bornhuetter-Ferguson Principle

were originally designed as deterministic algorithms) in a statistical setting. This means that all losses are interpreted as random variables which are either observable or not. In particular, the data represented in a run-off triangle are interpreted as realizations of observable losses and the outstanding or ultimate losses are non-observable (except for the initial accident year).

Once the statistical setting is accepted, the fundamental notion of a development pattern can be introduced. Although the notion of a development pattern is more or less present in many publications on loss reserving, its force with regard to the comparison of methods has been made evident only recently; see Radtke and Schmidt (2004) and Schmidt (2006).

Using the notion of a development pattern, we will show that the extended Bornhuetter-Ferguson method is not just a particular one among various methods of loss reserving but is a general one which contains many other methods as special cases and leads to the Bornhuetter-Ferguson principle, which consists of the simultaneous application of several versions of the extended Bornhuetter-Ferguson method to a given run-off triangle.

This paper extends the discussion of the extended Bornhuetter-Ferguson method which was started in some of the contributions in Radtke and Schmidt (2004) and was continued in Schmidt (2006; Section 4). The extension consists of

- the inclusion of development patterns which differ from the classical ones,
- a general discussion of the estimation of development patterns, and
- the embedding of quite recent methods of loss reserving into the extended Bornhuetter-Ferguson method.

In particular, we take into account the contributions of Mack (2006) and Panning (2006) in response to the CAS 2006 Reserves Call Paper Program.

This paper is organized as follows:

In Section 2, we introduce the general modeling of loss development data by a family of random variables representing the incremental or cumulative losses and the representation of the observable incremental or cumulative losses by a run-off triangle.

In Section 3, we introduce and study the central notion of a development pattern which turns out to be a powerful and unifying concept for the interpretation and comparison of several methods of loss reserving based on run-off triangles. We show that the notion of a development pattern can be expressed in several different but equivalent ways and that a development pattern can also be obtained on the basis of volume measures.

In Section 4, we study the problem of estimating the parameters of a development pattern.

In Section 5, we present the extended Bornhuetter-Ferguson method in its predictive form for the ultimate (cumulative) losses and we show that many other methods of loss reserving can be interpreted as special cases. This section is the central part of the present paper. The results of this section are summarized in a table which indicates that certain combinations of estimators of the development pattern and of the expected ultimate losses yield new methods of loss reserving which have not yet been considered in the literature.

In Section 6, we illustrate possible applications of the Bornhuetter-Ferguson principle by a numerical example. For a given run-off triangle, we use a variety of versions of the extended Bornhuetter-Ferguson method to compute predictors of the first year reserve and the total reserve. The realizations of these predictors are visualized in a two-dimensional plot which, when combined with actuarial judgment, may be used to determine best predictors and ranges; in addition, they may be used to compare the portfolio under consideration with a market portfolio or to check whether premiums are adequate or not.

In Section 7, we present proofs of two non-evident results used in Section 5.
2. Run-off triangles

We consider a portfolio of risks and we assume that each claim of the portfolio is settled either in the accident year or in the following \( n \) development years. The portfolio may be modeled either by incremental losses or by cumulative losses.

2.1. Incremental losses

To model a portfolio by incremental losses, we consider a family of random variables \( \{Z_{i,k}\}_{i,k \in \{0,1,\ldots,n\}} \) and we interpret the random variable \( Z_{i,k} \) as the loss of accident year \( i \) which is settled with a delay of \( k \) years and hence in development year \( k \) and in calendar year \( i+k \). We refer to \( Z_{i,k} \) as the incremental loss of accident year \( i \) and development year \( k \).

We assume that the incremental losses \( Z_{i,k} \) are observable for calendar years \( i+k \leq n \) and that they are non-observable for calendar years \( i+k \geq n+1 \). The observable incremental losses are represented by the following run-off triangle:

\[
\begin{array}{ccccccccc}
\text{Accident Year} & 0 & 1 & \cdots & k & \cdots & n-i & \cdots & n-1 & n \\
0 & Z_{0,0} & Z_{0,1} & \cdots & Z_{0,k} & \cdots & Z_{0,n-i} & \cdots & Z_{0,n-1} & Z_{0,n} \\
1 & Z_{1,0} & Z_{1,1} & \cdots & Z_{1,k} & \cdots & Z_{1,n-i} & \cdots & Z_{1,n-1} & \vdots \\
\vdots & \vdots & \vdots & \ddots & \vdots & \ddots & \vdots & \ddots & \vdots \\
\vdots & \vdots & \vdots & \ddots & Z_{i,0} & \cdots & Z_{i,k} & \cdots & Z_{i,n-i} \\
\vdots & \vdots & \vdots & \ddots & \vdots & \ddots & \vdots & \ddots & \vdots \\
n-k & Z_{n-k,0} & Z_{n-k,1} & \cdots & Z_{n-k,k} & \vdots & \vdots & \vdots \\
\vdots & \vdots & \vdots & \ddots & Z_{n-1,0} & \cdots & Z_{n-1,1} \\
n & Z_{n,0} & & & & & & & \\
\end{array}
\]

The problem is to predict the non-observable incremental losses.

2.2. Cumulative losses

To model a portfolio by cumulative losses, we consider a family of random variables \( \{S_{i,k}\}_{i,k \in \{0,1,\ldots,n\}} \) and we interpret the random variable \( S_{i,k} \) as the loss of accident year \( i \) which is settled with a delay of at most \( k \) years and hence not later than in development year \( k \). We refer to \( S_{i,k} \) as the cumulative loss of accident year \( i \) and development year \( k \), to \( S_{i,n-i} \) as a cumulative loss of the present calendar year \( n \) or as a current (cumulative) loss, and to \( S_{i,n} \) as an ultimate (cumulative) loss.

We assume that the cumulative losses \( S_{i,k} \) are observable for calendar years \( i+k \leq n \) and that they are non-observable for calendar years \( i+k \geq n+1 \). The observable cumulative losses are represented by the following run-off triangle:

\[
\begin{array}{ccccccccc}
\text{Accident Year} & 0 & 1 & \cdots & k & \cdots & n-i & \cdots & n-1 & n \\
0 & S_{0,0} & S_{0,1} & \cdots & S_{0,k} & \cdots & S_{0,n-i} & \cdots & S_{0,n-1} & S_{0,n} \\
1 & S_{1,0} & S_{1,1} & \cdots & S_{1,k} & \cdots & S_{1,n-i} & \cdots & S_{1,n-1} & \vdots \\
\vdots & \vdots & \vdots & \ddots & \vdots & \ddots & \vdots & \ddots & \vdots \\
i & S_{i,0} & S_{i,1} & \cdots & S_{i,k} & \cdots & S_{i,n-i} & \cdots & \vdots \\
\vdots & \vdots & \vdots & \ddots & \vdots & \ddots & \vdots & \ddots & \vdots \\
n-k & S_{n-k,0} & S_{n-k,1} & \cdots & S_{n-k,k} & \vdots & \vdots & \vdots \\
\vdots & \vdots & \vdots & \ddots & Z_{n-1,0} & \cdots & Z_{n-1,1} \\
n & S_{n,0} & & & & & & & \\
\end{array}
\]

The problem is to predict the non-observable cumulative losses.

2.3. Remarks

Of course, modeling a portfolio by incremental losses is equivalent to modeling a portfolio by cumulative losses:

- The cumulative losses are obtained from the incremental losses by letting

\[
S_{i,k} := \sum_{l=0}^{k} Z_{i,l}.
\]

Then the non-observable cumulative losses satisfy

\[
S_{i,k} = S_{i,n-i} + \sum_{l=n-i+1}^{k} Z_{i,l}.
\]
• The incremental losses are obtained from the cumulative losses by letting

\[ Z_{i,k} := \begin{cases} S_{i,0} & \text{if } k = 0 \\ S_{i,k} - S_{i,k-1} & \text{else} \end{cases} \]

In the sequel we shall switch between incremental and cumulative losses as necessary.

Correspondingly, prediction of non-observable incremental losses is equivalent to prediction of non-observable cumulative losses:

• If \( \{Z_{i,k}\}_{i \in \{0,1,\ldots,n\}, \; i+k \geq n+1} \) is a family of predictors of the non-observable incremental losses, then a family of predictors of the non-observable cumulative losses is obtained by letting

\[ \hat{S}_{i,n-i} := S_{i,n-i} + \sum_{l=n-i+1}^{k} Z_{i,l} . \]

• If \( \{\hat{Z}_{i,k}\}_{i \in \{0,1,\ldots,n\}, \; i+k \geq n+1} \) is a family of predictors of the non-observable cumulative losses, then a family of predictors of the non-observable incremental losses is obtained by letting

\[ \hat{Z}_{i,k} := \begin{cases} \hat{S}_{i,n-i+1} - S_{i,n-i} & \text{if } k = n-i+1 \\ \hat{S}_{i,k} - \hat{S}_{i,k-1} & \text{else} \end{cases} \]

For the ease of notation and to avoid the distinction of cases as in the previous definition, we shall also refer to \( Z_{i,n-i} \) and \( S_{i,n-i} \) as predictors of \( Z_{i,n-i} \) and \( S_{i,n-i} \), although these random variables are, of course, observable.

The enumeration of accident years and development years starting with 0 instead of 1 is widely but not yet generally accepted; see Stanard (1985), Taylor (2000), Radtke and Schmidt (2004), Panning (2006) and the publications of the present authors. It is useful for several reasons:

• For losses which are settled within the accident year, the delay of settlement is 0. It is therefore natural to start the enumeration of development years with 0.
• Using the enumeration of development years also for accident years implies that the incremental or cumulative loss of accident year \( i \) and development year \( k \) is observable if and only if \( i + k \leq n \). In particular, the current losses \( S_{i,n-i} \) are those of the present calendar year \( n \) and are crucial in most methods of loss reserving.

After all, the notation used here simplifies mathematical formulas.

3. Development patterns

The use of run-off triangles in loss reserving can be justified only if it is assumed that the development of the losses of every accident year follows a development pattern which is common to all accident years. This vague idea of a development pattern can be formalized in various ways.

In the present section we consider three classical development patterns, which are formally distinct but nevertheless similar and can easily be converted into each other, and we also introduce alternative development patterns. The variety of development patterns is nevertheless needed since each of these development patterns occurs as a natural primitive model for some of the methods of loss reserving to be discussed in Section 5 or as a part of certain more sophisticated models; see Schmidt (2006).

The assumption of an underlying development pattern can be viewed as a primitive stochastic model and provides the key to the comparison of various methods of loss reserving.

3.1. Incremental quotas

A vector \( \vartheta = (\vartheta_0, \vartheta_1, \ldots, \vartheta_n) \) of parameters (with \( \sum_{l=0}^{n} \vartheta_l = 1 \)) is said to be a development pattern for incremental quotas if the identity

\[ \vartheta_k = \frac{E[Z_{i,k}]}{E[S_{i,0}]} \]

holds for all \( k \in \{0,1,\ldots,n\} \) and for all \( i \in \{0,1,\ldots,n\} \).
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Thus, a development pattern for incremental quotas exists if, and only if, for every development year \( k \in \{0,1,\ldots,n\} \) the individual incremental quotas
\[
\vartheta_{i,k} := \frac{E[Z_{i,k}]}{E[S_{i,n}]} \]
are identical for all accident years.

In the case of a run-off triangle for paid losses or claim counts it is usually reasonable to assume in addition that \( \vartheta_k > 0 \) holds for all \( k \in \{0,1,\ldots,n\} \). In the case of incurred losses, however, this additional assumption may be inappropriate since, due to conservative loss reserving, the expected incremental losses of development years \( k \in \{1,\ldots,n\} \) may be negative.

3.2. Cumulative quotas

A vector \( \gamma = (\gamma_0, \gamma_1, \ldots, \gamma_n) \) of parameters (with \( \gamma_n = 1 \)) is said to be a development pattern for cumulative quotas if the identity
\[
\gamma_k = \frac{E[S_{i,k}]}{E[S_{i,n}]} \]
holds for all \( k \in \{0,1,\ldots,n\} \) and for all \( i \in \{0,1,\ldots,n\} \).

Thus, a development pattern for cumulative quotas exists if, and only if, for every development year \( k \in \{0,1,\ldots,n\} \) the individual cumulative quotas
\[
\gamma_{i,k} := \frac{E[S_{i,k}]}{E[S_{i,n}]} \]
are identical for all accident years.

In the case of a run-off triangle for paid losses or claim counts, it is usually reasonable to assume in addition that \( 0 < \gamma_0 < \gamma_1 < \cdots < \gamma_n \).

The development patterns for cumulative quotas and for incremental quotas can be converted into each other:

- If \( \varphi \) is a development pattern for cumulative quotas, then a development pattern \( \vartheta \) for incremental quotas is obtained by letting
  \[
  \vartheta_k := \begin{cases} 
  \gamma_0 & \text{if } k = 0 \\
  \gamma_k - \gamma_{k-1} & \text{else} 
  \end{cases} 
  \]
- If \( \vartheta \) is a development pattern for incremental quotas, then a development pattern \( \gamma \) for cumulative quotas is obtained by letting
  \[
  \gamma_k := \sum_{l=0}^{k} \vartheta_l. 
  \]

Furthermore, the condition \( 0 < \gamma_0 < \gamma_1 < \cdots < \gamma_n \) is fulfilled if, and only if, \( \vartheta_k > 0 \) holds for all \( k \in \{0,1,\ldots,n\} \).

3.3. Factors

A vector \( \varphi = (\varphi_1, \ldots, \varphi_n) \) of parameters is said to be a development pattern for factors if the identity
\[
\varphi_k = \frac{E[S_{i,k}]}{E[S_{i,k-1}]} 
\]
holds for all \( k \in \{1,\ldots,n\} \) and for all \( i \in \{0,1,\ldots,n\} \).

Thus, a development pattern for factors exists if, and only if, for every development year \( k \in \{1,\ldots,n\} \) the individual factors
\[
\varphi_{i,k} := \frac{E[S_{i,k}]}{E[S_{i,k-1}]} 
\]
are identical for all accident years.

In the case of a run-off triangle for paid losses or claim counts, it is usually reasonable to assume in addition that \( \varphi_k > 1 \) holds for all \( k \in \{1,\ldots,n\} \).

The development patterns for factors and for cumulative quotas can be converted into each other:

- If \( \varphi \) is a development pattern for factors, then a development pattern \( \gamma \) for cumulative quotas is obtained by letting
  \[
  \gamma_k := \prod_{l=k+1}^{n} \frac{1}{\varphi_l}. 
  \]
- If \( \gamma \) is a development pattern for cumulative quotas, then a development pattern \( \varphi \) for factors is obtained by letting
  \[
  \varphi_k := \frac{\gamma_k}{\gamma_{k-1}}. 
  \]
Furthermore, $\varphi_k > 1$ holds for all $k \in \{1, \ldots, n\}$ if, and only if, the condition $\gamma_0 < \gamma_1 < \cdots < \gamma_n$ is fulfilled.

Combining the previous result and that of the preceding subsection, it is evident that also the development patterns for factors and for incremental quotas can be converted into each other.

### 3.4. Incremental ratios

The classical development patterns considered before are quite familiar and have been shown to be equivalent.

An alternative development pattern can be distilled from the paper by Panning (2006) which was written in response to the CAS 2006 Reserves Call Paper Program:

A vector $\beta = (\beta_0, \beta_1, \ldots, \beta_n)$ of parameters (with $\beta_0 = 1$) is said to be a development pattern for incremental ratios if the identity

$$
\beta_k = \frac{E[Z_{i,k}]}{E[Z_{i,0}]}
$$

holds for all $k \in \{0, 1, \ldots, n\}$ and for all $i \in \{0, 1, \ldots, n\}$.

Thus, a development pattern for incremental ratios exists if, and only if, for every development year $k \in \{0, 1, \ldots, n\}$ the individual incremental ratios

$$
\beta_{i,k} := \frac{E[Z_{i,k}]}{E[Z_{i,0}]}
$$

are identical for all accident years.

In the case of a run-off triangle for paid losses or claim counts it is usually reasonable to assume in addition that $\beta_k > 0$ holds for all $k \in \{0, 1, \ldots, n\}$.

The development patterns for incremental ratios and for incremental quotas can be converted into each other:

- If $\beta$ is a development pattern for incremental ratios, then a development pattern $\vartheta$ for incremental quotas is obtained by letting

$$
\vartheta_k := \frac{\beta_k}{\sum_{l=0}^{n} \beta_l}.
$$

Moreover, the development patterns for incremental ratios and for cumulative quotas can be converted into each other as well:

- If $\beta$ is a development pattern for incremental ratios, then a development pattern $\gamma$ for cumulative quotas is obtained by letting

$$
\gamma_k := \frac{\sum_{l=0}^{k} \beta_l}{\sum_{l=0}^{n} \beta_l}.
$$

Furthermore, $\beta_k > 0$ holds for all $k \in \{0, 1, \ldots, n\}$ if and only if $\vartheta_k > 0$ holds for all $k \in \{0, 1, \ldots, n\}$, and this is the case if and only if $0 < \gamma_0 < \gamma_1 < \cdots < \gamma_n$.

### 3.5. Incremental loss ratios

The development pattern considered so far are completely determined by expected incremental or cumulative losses. However, if a vector $\pi = (\pi_0, \pi_1, \ldots, \pi_n)$ of known volume measures (like premiums or the number of contracts) is given, then another development pattern can be defined which also depends on the volume measure $\pi$.

A vector $\zeta(\pi) = (\zeta_0(\pi), \zeta_1(\pi), \ldots, \zeta_n(\pi))$ of parameters is said to be a development pattern for incremental loss ratios if the identity

$$
\zeta_k(\pi) = E \left[ \frac{Z_{i,k}}{\pi_i} \right]
$$

holds for all $k \in \{0, 1, \ldots, n\}$ and for all $i \in \{0, 1, \ldots, n\}$. 

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Thus, a development pattern for incremental loss ratios exists if, and only if, for every development year $k \in \{0, 1, \ldots, n\}$ the \textit{individual incremental loss ratios}
\[ \zeta_{i,k}(\pi) := E \left[ \frac{Z_{i,k}}{\pi_i} \right] \]
are identical for all accident years.

In the case of a run-off triangle for paid losses or claim counts it is usually reasonable to assume in addition that $\zeta_k(\pi) > 0$ holds for all $k \in \{0, 1, \ldots, n\}$.

If $\zeta(\pi)$ is a development pattern for incremental loss ratios, then
- a development pattern $\vartheta(\pi)$ for incremental quotas is obtained by letting
  \[ \vartheta_k(\pi) := \frac{\zeta_k(\pi)}{\sum_{l=0}^{n} \zeta_l(\pi)} \]
and
- a development pattern $\gamma(\pi)$ for cumulative quotas is obtained by letting
  \[ \gamma_k(\pi) := \frac{\sum_{l=0}^{k} \zeta_l(\pi)}{\sum_{l=0}^{n} \zeta_l(\pi)} \]
These definitions are entirely analogous to those used in the case of a development pattern for incremental ratios.

\section*{3.6. Remarks}

In the case of a run-off triangle for paid losses or claim counts, the intuitive interpretation of the development patterns of incremental or cumulative quotas would be their interpretation as incremental or cumulative probabilities. This interpretation is helpful, but it is not quite correct since the parameters of these development patterns are defined in terms of \textit{quotients of expectations} instead of \textit{expectations of quotients}; as it is well-known, these quantities are in general distinct.

One may thus argue that the definitions of development patterns are inconvenient since they do not exactly correspond to intuition. The development patterns defined in terms of quotients of expectations are nevertheless reasonable since they are all equivalent in the sense that they can be converted into each other (which would be impossible for certain development patterns defined in terms of expectations of quotients). Due to this equivalence, the development patterns presented here provide a powerful and unifying concept for the interpretation and comparison of several methods of loss reserving.

Quite generally, alternative development patterns can be derived from the classical ones by interchanging the roles of incremental and cumulative losses and/or the roles of the initial and the ultimate development year. In this sense, the development pattern for incremental ratios corresponds to the development pattern for cumulative quotas.

\section*{4. Estimation of development patterns}

For each of the methods of loss reserving to be discussed in Section 5, the predictors of the ultimate losses can be justified by the assumption that a development pattern exists. This is due to the fact that, in either case, the predictors can be expressed in terms of certain estimators of the parameters of the development pattern for cumulative quotas.

Quite generally, estimation of the development pattern can be based on one or both of the following different sources of information:
- \textbf{Internal information:} This is any information which is completely contained in the run-off triangle of the portfolio under consideration.
- \textbf{External information:} This is any information which is completely independent of the run-off triangle of the portfolio under consideration. External information could be obtained, e.g., from market statistics or from other portfolios which are judged to be similar to the given one; also, volume measures (like premiums or
the number of contracts) for the given portfolio may also be combined, in which case estimation is based on mixed information.

Of course, these different sources of information may also be combined, in which case estimation is based on mixed information.

It is possible to develop a general theory on the estimation of the parameters of a development pattern, but this would exceed the scope of the present paper. Instead, we shall confine ourselves to the presentation of the three types of estimators which will be needed in Section 5. The first two of these estimators are entirely based on internal information while the third one is based on mixed information. Nevertheless, the similarity of these three types of estimators indicates a general principle of estimation which can be applied to any development pattern.

The estimators presented below are based on the development patterns for factors, incremental ratios, and incremental loss ratios, respectively. Since each of these development patterns can be converted into a development pattern for cumulative quotas, as shown in Section 3, the same conversion formulas will be used to convert these estimators into estimators of the parameters of the corresponding development pattern for cumulative quotas.

4.1. Estimation from empirical individual factors

At the first glance, there is little hope to estimate the parameters of the development patterns for incremental or cumulative quotas since the only obvious estimators of $\varphi_k$ and $\gamma_k$ are the empirical individual incremental quotas $Z_{0,k}/S_{0,n}$ and the empirical individual cumulative quotas $S_{0,k}/S_{0,n}$, respectively. Fortunately, the situation is quite different for the development pattern for factors:

Assume that $\varphi = (\varphi_1, \ldots, \varphi_n)$ is a development pattern for factors. Then, for every development year $k \in \{1, \ldots, n\}$, each of the empirical individual factors or link ratios

$$\hat{\varphi}_{i,k} := \frac{S_{i,k}}{S_{i,k-1}}$$

with $i \in \{0, 1, \ldots, n - k\}$ is a reasonable estimator of $\varphi_k$, and this is also true for every weighted mean

$$\hat{\varphi}_k := \sum_{j=0}^{n-k} W_{j,k} \hat{\varphi}_{j,k}$$

with random variables (or constants) satisfying $\sum_{j=0}^{n-k} W_{j,k} = 1$. The most prominent estimator of this large family is the chain-ladder factor

$$\hat{\varphi}^{\text{CL}}_k := \frac{\sum_{j=0}^{n-k} S_{j,k}}{\sum_{j=0}^{n-k} S_{j,k-1}} \sum_{j=0}^{n-k} \frac{S_{j,k-1}}{\sum_{h=0}^{n-k} S_{h,k-1}} \hat{\varphi}_{j,k},$$

which is used in the chain-ladder method. We denote by

$$\hat{\varphi}^{\text{CL}} := (\hat{\varphi}_1^{\text{CL}}, \ldots, \hat{\varphi}_n^{\text{CL}})$$

the random vector consisting of all chain-ladder factors.

Due to the correspondence between the development patterns for factors and for cumulative quotas, it is clear that in the same way estimators of factors can be converted into estimators of cumulative quotas. In particular, the chain-ladder quotas

$$\hat{\gamma}^{\text{CL}}_k := \prod_{l=k+1}^{n} \frac{1}{\hat{\varphi}^{\text{CL}}_l}$$

serve as estimators of the cumulative quotas

$$\gamma_k := \prod_{l=k+1}^{n} \frac{1}{\varphi_l}.$$

We denote by

$$\hat{\gamma}^{\text{CL}} := (\hat{\gamma}^{\text{CL}}_0, \hat{\gamma}^{\text{CL}}_1, \ldots, \hat{\gamma}^{\text{CL}}_n)$$

the random vector consisting of all chain-ladder quotas.

We remark that the chain-ladder quotas are entirely based on internal information.
4.2. Estimation from empirical individual incremental ratios

Assume that \( \beta = (\beta_0, \beta_1, \ldots, \beta_n) \) is a development pattern for incremental ratios. Then, for every development year \( k \in \{0, 1, \ldots, n\} \), each of the empirical individual incremental ratios

\[
\hat{\beta}_{j,k} := \frac{Z_{j,k}}{Z_{j,0}}
\]

with \( i \in \{0, 1, \ldots, n - k\} \) is a reasonable estimator of \( \beta_k \), and this is also true for every weighted mean

\[
\hat{\beta}_k := \sum_{j=0}^{n-k} W_{j,k} \hat{\beta}_{j,k}
\]

with random variables (or constants) satisfying \( \sum_{j=0}^{n-k} W_{j,k} = 1 \). An example of this large family is the Panning ratio

\[
\hat{\beta}_{Panning,k} := \frac{\sum_{j=0}^{n-k} Z_{j,k} Z_{j,0}}{\sum_{j=0}^{n-k} Z_{j,0} Z_{j,0}^2} = \frac{1}{\sum_{j=0}^{n-k} Z_{j,0} Z_{j,0}^2} \hat{\beta}_{j,k}
\]

which is used in Panning’s method. We denote by

\[
\hat{\beta}_{Panning} := (\hat{\beta}_{0, Panning}, \hat{\beta}_{1, Panning}, \ldots, \hat{\beta}_{n, Panning})
\]

the random vector consisting of all Panning ratios.

Due to the correspondence between the development patterns for incremental ratios and for cumulative quotas, it is clear that in the same way estimators of incremental ratios can be converted into estimators of cumulative quotas. In particular, the Panning quotas

\[
\gamma_k := \frac{\sum_{l=0}^{k} \hat{\beta}_{Panning}}{\sum_{l=0}^{n} \hat{\beta}_{Panning}}
\]

serve as estimators of the cumulative quotas

\[
\gamma_k := \frac{\sum_{l=0}^{k} \hat{\beta}_l}{\sum_{l=0}^{n} \hat{\beta}_l}
\]

We denote by

\[
\hat{\gamma}_{Panning} := (\hat{\gamma}_0, \hat{\gamma}_1, \ldots, \hat{\gamma}_n)
\]

the random vector consisting of all Panning quotas.

We remark that the Panning quotas are entirely based on internal information.

4.3. Estimation from empirical individual incremental loss ratios

Assume that \( \pi = (\pi_0, \pi_1, \ldots, \pi_n) \) is a vector of known volume measures and that \( \zeta(\pi) = (\zeta_0(\pi), \zeta_1(\pi), \ldots, \zeta_n(\pi)) \) is a development pattern for loss ratios. Then, for every development year \( k \in \{0, 1, \ldots, n\} \), each of the empirical individual incremental loss ratios

\[
\hat{\zeta}_{i,k} := \frac{Z_{i,k}}{\pi_k}
\]

with \( i \in \{0, 1, \ldots, n - k\} \) is a reasonable estimator of \( \zeta_k(\pi) \), and this is also true for every weighted mean

\[
\hat{\zeta}_k := \sum_{j=0}^{n-k} W_{j,k} \hat{\zeta}_{j,k}(\pi)
\]

with random variables (or constants) satisfying \( \sum_{j=0}^{n-k} W_{j,k} = 1 \). The most prominent estimator of this large family is the additive loss ratio

\[
\hat{\zeta}_{AD}(\pi) := \frac{\sum_{j=0}^{n-k} Z_{j,k}}{\sum_{j=0}^{n-k} \pi_j} = \frac{1}{\sum_{j=0}^{n-k} \pi_j} \hat{\zeta}_{j,k}(\pi),
\]

which is used in the additive method. We denote by

\[
\hat{\zeta}_{AD}(\pi) := (\hat{\zeta}_0, \hat{\zeta}_1, \ldots, \hat{\zeta}_n)
\]

the random vector consisting of all additive loss ratios.

In view of the transformation of a development pattern for incremental loss ratios into a development pattern for cumulative quotas, it is clear that in the same way estimators of incremental loss ratios can be converted into estimators of cumulative quotas. In particular, the additive quotas

\[
\hat{\gamma}_k := \frac{\sum_{l=0}^{k} \hat{\zeta}_{AD}(\pi)}{\sum_{l=0}^{n} \hat{\zeta}_{AD}(\pi)}
\]
serve as estimators of the cumulative quotas

\[ \gamma_k(\pi) := \frac{\sum_{l=0}^{k} \zeta_l(\pi)}{\sum_{l=0}^{n} \zeta_l(\pi)} \]

We denote by

\[ \hat{\gamma}^{\text{AD}}(\pi) := (\hat{\gamma}_0^{\text{AD}}(\pi), \hat{\gamma}_1^{\text{AD}}(\pi), \ldots, \hat{\gamma}_n^{\text{AD}}(\pi)) \]

the random vector consisting of all additive quotas.

We remark that the additive quotas are based on mixed information, since they involve the internal information provided by the run-off triangle and the external information provided by the volume measure.

4.4. Remarks

The use of weighted means in estimating the parameters of a development pattern can be justified in a linear model with uncorrelated dependent variables and suitably chosen variances. For example,

- the chain-ladder factors can be justified in the chain-ladder model of Mack and Schnaus [see Mack (1994), Schmidt and Schnaus (1996), Radtke and Schmidt (2004), and Schmidt (2006)],
- the Panning ratios can be justified in the model of Panning (2006), and
- the additive loss ratios can be justified in the linear model of Mack [see Mack (1991), Radtke and Schmidt (2004), and Schmidt (2006)].

Each of these models is a linear model with a particular assumption on the variances and the afore-mentioned estimators have the Gauss-Markov property. If, however, in any of these models the assumption on the variances would be changed, then the Gauss-Markov estimators of the parameters would be weighted means which are distinct from the chain-ladder factors, the Panning ratios, or the additive loss ratios, respectively.

5. Prediction of ultimate losses

The present section provides a unifying presentation of the most important methods of loss reserving. The starting point is an extension of the Bornhuetter-Ferguson method which is closely related to the notion of a development pattern for cumulative quotas and turns out to be a unifying principle under which various other methods of loss reserving can be subsumed.

5.1. Extended Bornhuetter-Ferguson method

The extended Bornhuetter-Ferguson method is based on the assumption that there exist vectors \( \alpha = (\alpha_0, \alpha_1, \ldots, \alpha_n) \) and \( \gamma = (\gamma_0, \gamma_1, \ldots, \gamma_n) \) of parameters (with \( \gamma_n = 1 \)) such that the identity

\[ E[S_{i,k}] = \gamma_k \alpha_i \]

holds for all \( k \in \{0, 1, \ldots, n\} \) and for all \( i \in \{0, 1, \ldots, n\} \). Then we have

\[ E[S_{i,n}] = \alpha_i \]

and hence

\[ \gamma_k = \frac{E[S_{i,k}]}{E[S_{i,n}]}, \]

which means that \( \gamma \) is a development pattern for cumulative quotas.

The extended Bornhuetter-Ferguson method is also based on the additional assumption that a vector \( \hat{\gamma} = (\hat{\gamma}_0, \hat{\gamma}_1, \ldots, \hat{\gamma}_n) \) of prior estimators of the cumulative quotas with \( \hat{\gamma}_n = 1 \) and a vector \( \hat{\alpha} = (\hat{\alpha}_0, \hat{\alpha}_1, \ldots, \hat{\alpha}_n) \) of prior estimators of the expected ultimate losses are given. As already indicated in Section 4, the

---

2The term prior in connection with the estimators of the cumulative quotas and the expected ultimate losses needs some explanation: It is used here only to indicate that these estimators are needed before the computation of the Bornhuetter-Ferguson predictors. Of course, the estimators of the expected ultimate losses could also be regarded as preliminary predictors of the ultimate losses, but this point is minor, and there will be no update of the estimators of the cumulative quotas.
prior estimators of the cumulative quotas can be obtained from internal, external, or mixed information. This is, of course, also true for the prior estimators of the expected ultimate losses.

The Bornhuetter-Ferguson predictors of the cumulative losses \( S_{i,k} \) with \( i + k \geq n \) are defined as

\[
\hat{S}_{i,k}^\text{BF}(\gamma, \alpha) := S_{i,n-i} + (\hat{\gamma}_k - \hat{\gamma}_{n-i})\hat{\alpha}_i.
\]

The definition of the Bornhuetter-Ferguson predictors reminds us of the identity

\[
E[S_{i,k}] = E[S_{i,n-i}] + (\hat{\gamma}_k - \hat{\gamma}_{n-i})\alpha_i,
\]

which is a consequence of the model assumption. We denote by

\[
\hat{S}_{i,k}^\text{BF}(\gamma, \alpha) := (\hat{S}_{i,k}^\text{BF}(\gamma, \alpha))_{i,k \in \{0,1,...,n\}, \ i+k \geq n}
\]

the triangle of all Bornhuetter-Ferguson predictors.

Taking the difference between the Bornhuetter-Ferguson predictors and the current losses yields

\[
\hat{S}_{i,k}^\text{BF}(\gamma, \alpha) - S_{i,n-i} = (\hat{\gamma}_k - \hat{\gamma}_{n-i})\hat{\alpha}_i.
\]

In the case \( k = n \) this yields

\[
\hat{S}_{i,n}^\text{BF}(\gamma, \alpha) - S_{i,n-i} = (1 - \hat{\gamma}_{n-i})\hat{\alpha}_i,
\]

which is a predictor of the reserve \( S_{i,n} - S_{i,n-i} \) of accident year \( i \) and has the shape of the reserve predictors proposed by Bornhuetter and Ferguson (1972). However, in the original form of the Bornhuetter-Ferguson method it is assumed that the prior estimators of the expected ultimate losses are based on premiums and expected loss ratios while those of the development pattern are obtained from the run-off triangle. Both assumptions are dropped in the extended Bornhuetter-Ferguson method, and this is the key to arranging various methods of loss reserving, which at the first glance have little in common, under a common umbrella.

### 5.2. Iterated Bornhuetter-Ferguson method

In the case where the current losses are judged to be reliable, it may be desirable to modify the Bornhuetter-Ferguson predictors in order to strengthen the weight of the current losses and to reduce that of the prior estimators of the expected ultimate losses. This goal can be achieved by iteration.

For example, if on the right-hand side of the formula defining the Bornhuetter-Ferguson predictors the prior estimators \( \hat{\alpha}_i \) are replaced by the Bornhuetter-Ferguson predictors \( \hat{S}_{i,n}^\text{BF} \), then the resulting predictors of the cumulative losses \( S_{i,k} \) with \( i + k \geq n \) are the Benktander-Hovinen predictors

\[
\hat{S}_{i,k}^\text{BH}(\gamma, \alpha) := S_{i,n-i} + (\hat{\gamma}_k - \hat{\gamma}_{n-i})\hat{\alpha}_i^m(\gamma, \alpha),
\]

which, in the case \( \hat{\gamma}_0 < \hat{\gamma}_1 < \cdots < \hat{\gamma}_n = 1 \), increase the weight of the current losses and reduce that of the prior estimators of the expected ultimate losses.

More generally, the iterated Bornhuetter-Ferguson predictors of order \( m \in \mathbb{N}_0 \) of the cumulative losses \( S_{i,k} \) with \( i + k \geq n \) are defined by letting

\[
\hat{S}_{i,k}^m(\gamma, \alpha) := \begin{cases} 
S_{i,n-i} + (\hat{\gamma}_k - \hat{\gamma}_{n-i})\alpha_i & \text{if } m = 0 \\
S_{i,n-i} + (\hat{\gamma}_k - \hat{\gamma}_{n-i})\hat{S}_{i,n}^{(m-1)}(\gamma, \alpha) & \text{else}
\end{cases}
\]

Then we have \( \hat{S}_{i,k}^{(0)}(\gamma, \alpha) = \hat{S}_{i,k}^\text{BF}(\gamma, \alpha) \) and \( \hat{S}_{i,k}^{(1)}(\gamma, \alpha) = \hat{S}_{i,k}^\text{BH}(\gamma, \alpha) \). We denote by

\[
\hat{S}_{i,k}^m(\gamma, \alpha) := (\hat{S}_{i,k}^m(\gamma, \alpha))_{i,k \in \{0,1,...,n\}, \ i+k \geq n}
\]

the triangle of all Bornhuetter-Ferguson predictors of order \( m \). Letting

\[
\hat{\alpha}_i^m(\gamma, \alpha) := \begin{cases} 
\hat{\alpha}_i & \text{if } m = 0 \\
\hat{S}_{i,n}^{(m-1)} & \text{else}
\end{cases}
\]

the iterated Bornhuetter-Ferguson predictors can be written as

\[
\hat{S}_{i,k}^m(\gamma, \alpha) = S_{i,n-i} + (\hat{\gamma}_k - \hat{\gamma}_{n-i})\hat{\alpha}_i^m(\gamma, \alpha)
\]

---

3This interpretation of the method proposed by Benktander (1976), which in fact is more general, follows Mack (2000). The Benktander method was rediscovered by Hovinen (1981) and a related paper is that of Neuhaus (1992).

4The iterated Bornhuetter-Ferguson method is due to Mack (2000). With regard to terminology, it should be noted that in Mack (2000) the loss-development predictors presented in Subsection 5.3 are referred to as chain-ladder predictors.
and with
\[ \hat{\alpha}^{(m)}(\hat{\gamma}, \hat{\alpha}) := (\hat{\alpha}_0^{(m)}(\hat{\gamma}, \hat{\alpha}), \hat{\alpha}_1^{(m)}(\hat{\gamma}, \hat{\alpha}), \ldots, \hat{\alpha}_n^{(m)}(\hat{\gamma}, \hat{\alpha})) \]
we obtain
\[ \hat{S}^{(m)}(\hat{\gamma}, \hat{\alpha}) = \hat{S}^{BF}(\hat{\alpha}^{(m)}(\hat{\gamma}, \hat{\alpha}), \hat{\gamma}). \]
Therefore, the iterated Bornhuetter-Ferguson method of order \( m \) with respect to \( \hat{\gamma} \) and \( \hat{\alpha} \) is nothing else than the extended Bornhuetter-Ferguson method with respect to \( \hat{\gamma} \) and \( \hat{\alpha}^{(m)}(\hat{\gamma}, \hat{\alpha}) \).

### 5.3. Loss-development method

The **loss-development method**\(^5\) is based on the assumption that there exists a vector \( \gamma = (\gamma_0, \gamma_1, \ldots, \gamma_n) \) of parameters (with \( \gamma_n = 1 \)) such that the identity
\[ \gamma_k = E[S_{i,k}] \]
holds for all \( k \in \{0, 1, \ldots, n\} \) and for all \( i \in \{0, 1, \ldots, n\} \). Then \( \gamma \) is a development pattern for cumulative quotas.

The loss-development method is also based on the additional assumption that a vector
\[ \hat{\gamma} = (\hat{\gamma}_0, \hat{\gamma}_1, \ldots, \hat{\gamma}_n) \]
of prior estimators of the cumulative quotas with \( \hat{\gamma}_n = 1 \) is given.

The **loss-development predictors** of the cumulative losses \( S_{i,k} \) with \( i + k \geq n \) are defined as
\[ \hat{S}_{i,k}^{LD}(\hat{\gamma}) := \frac{\hat{\gamma}_k S_{i,n-i}}{\gamma_{n-i}}. \]
The definition of the loss-development predictors reminds of the identity
\[ E[S_{i,k}] = \gamma_k \frac{E[S_{i,n-i}]}{\gamma_{n-i}}, \]
which is a consequence of the model assumption. We denote by
\[ \hat{S}_{i,k}^{LD}(\hat{\gamma}) := (\hat{S}_{i,k}^{LD}(\hat{\gamma}))_{i,k \in \{0, 1, \ldots, n\}, i+k \geq n} \]
the triangle of all loss-development predictors. It is immediate from the definition that the loss-development predictors satisfy
\[ \hat{S}_{i,k}^{LD}(\hat{\gamma}) = S_{i,n-i} + (\hat{\gamma}_k - \hat{\gamma}_{n-i}) \hat{S}_{i,n}^{LD}(\hat{\gamma}). \]
Letting
\[ \hat{\alpha}_i^{LD}(\hat{\gamma}) := \hat{S}_{i,n}^{LD}(\hat{\gamma}) \]
the loss-development predictors can be written as
\[ \hat{S}_{i,k}^{LD}(\hat{\gamma}) = S_{i,n-i} + (\hat{\gamma}_k - \hat{\gamma}_{n-i}) \hat{\alpha}_i^{LD}(\hat{\gamma}) \]
and with
\[ \hat{\alpha}^{LD}(\hat{\gamma}) := (\hat{\alpha}_0^{LD}(\hat{\gamma}), \hat{\alpha}_1^{LD}(\hat{\gamma}), \ldots, \hat{\alpha}_n^{LD}(\hat{\gamma})) \]
we obtain
\[ \hat{S}^{LD}(\hat{\gamma}) = \hat{S}^{BF}(\hat{\gamma}, \hat{\alpha}^{LD}(\hat{\gamma})). \]
Therefore, the loss-development method with respect to \( \hat{\gamma} \) is nothing else than the extended Bornhuetter-Ferguson method with respect to \( \hat{\gamma} \) and \( \hat{\alpha}^{LD}(\hat{\gamma}) \).

Moreover, it has been shown by Mack (2000) that the loss-development predictors are the limits of the iterated Bornhuetter-Ferguson predictors; see also Radtke and Schmidt (2004), or Schmidt (2006).

### 5.4. Chain-ladder method

The **chain-ladder method** is based on the assumption that there exists a vector \( \varphi = (\varphi_1, \ldots, \varphi_n) \) of parameters such that the identity
\[ \varphi_k = E[S_{i,k}] \]
holds for all \( k \in \{1, \ldots, n\} \) and for all \( i \in \{0, 1, \ldots, n\} \). Then \( \varphi \) is a development pattern for factors.

The **chain-ladder predictors** of the cumulative losses \( S_{i,k} \) with \( i + k \geq n \) are defined as
\[ \hat{S}_{i,k}^{CL} := S_{i,n-i} \prod_{l=n-i+1}^{k} \varphi_l^{CL}, \]
where
\[ \hat{\varphi}_k := \frac{\sum_{j=0}^{n-k} S_{j,k}}{\sum_{j=0}^{n-k} S_{j,k-1}} \]
is the chain-ladder factor introduced in Section 4. The definition of the chain-ladder predictors reminds us of the identity
\[ E[S_{i,k}] = E[S_{i,n-i}] \prod_{l=n-i+1}^{k} \varphi_l, \]
which is a consequence of the model assumption. We denote by
\[ \hat{S}^{CL} := (\hat{S}_{i,k})_{i,k \in \{0,1,\ldots,n\}}, \quad i+k \geq n \]
the triangle of all chain-ladder predictors. Since
\[ \hat{\gamma}_{k}^{CL} = \prod_{i=k+1}^{n} \frac{1}{\hat{\varphi}_l^{CL}}, \]
the chain-ladder predictors can be written as
\[ \hat{S}_{i,k}^{CL} = \hat{\gamma}_{k}^{CL} S_{i,n-i} \frac{\hat{\varphi}_{n-i}^{CL}}{\hat{\varphi}_{n-i}^{CL}}. \]
We thus obtain
\[ \hat{S}^{CL} = \hat{S}^{LD}(\hat{\gamma}^{CL}) \]
and hence, using the result of the previous subsection,
\[ \hat{S}^{CL} = \hat{S}^{BF}(\hat{\gamma}^{CL}, \hat{\alpha}^{LD}(\hat{\gamma}^{CL})). \]
Because of these two identities, the chain-ladder method coincides with the loss-development method with respect to \( \hat{\gamma}^{CL} \) and is nothing else than the extended Bornhuetter-Ferguson method with respect to \( \hat{\gamma}^{CL} \) and \( \hat{\alpha}^{LD}(\hat{\gamma}^{CL}) \).

We recall that the chain-ladder factors can be written in the form
\[ \hat{\varphi}_k = \sum_{j=0}^{n-k} \frac{S_{j,k-1}}{S_{j,k}} \hat{\varphi}_{j,k}. \]
Therefore, the chain-ladder method can be modified by replacing the chain-ladder factors \( \hat{\varphi}_k \) by any other estimators of the form
\[ \hat{\varphi}_k = \sum_{j=0}^{n-k} W_{j,k} \hat{\varphi}_{j,k} \]
with random variables (or constants) satisfying \( \sum_{j=0}^{n-k} W_{j,k} = 1 \) for all \( k \in \{1, \ldots, n\} \). Every such modification yields a new development pattern of factors \( \hat{\varphi} \) and hence a new development pattern of cumulative quotas \( \hat{\gamma} \) such that the above identities for the chain-ladder predictors remain valid for the modified chain-ladder predictors with \( \hat{\varphi} \) and \( \hat{\gamma} \) in the place of \( \hat{\varphi}^{CL} \) and \( \hat{\gamma}^{CL} \), respectively. Every such modification of the chain-ladder method is a special case of the loss-development method.

5.5. Cape Cod method

The Cape Cod method is based on the assumption that there exists
- a vector \( \gamma = (\gamma_0, \gamma_1, \ldots, \gamma_n) \) of parameters (with \( \gamma_n = 1 \)) such that the identity
\[ \gamma_k = \frac{E[S_{i,k}]}{E[S_{i,n}]} \]
holds for all \( k \in \{0,1,\ldots,n\} \) and for all \( i \in \{0,1,\ldots,n\} \),
- a vector \( \pi = (\pi_0, \pi_1, \ldots, \pi_n) \) of known volume measures, and
- a parameter \( \kappa \) such that the identity
\[ \kappa = E \left[ \frac{S_{i,n}}{\pi_i} \right] \]
holds for all \( i \in \{0,1,\ldots,n\} \).

Then \( \gamma \) is a development pattern for cumulative quotas, the last assumption means that the individual ultimate loss ratios
\[ \kappa_i := E \left[ \frac{S_{i,n}}{\pi_i} \right] \]
are identical for all accident years, and the parameter \( \kappa \) is said to be the ultimate loss ratio (common to all accident years).

The Cape Cod method is also based on the additional assumption that a vector
\[ \hat{\gamma} = (\hat{\gamma}_0, \hat{\gamma}_1, \ldots, \hat{\gamma}_n) \]
of prior estimators of the cumulative quotas with \( \hat{\gamma}_n = 1 \) is given.

The Cape Cod predictors\(^6\) of the cumulative losses \( S_{i,k} \) with \( i + k \geq n \) are defined as

\[
\hat{S}^{CC}_{i,k}(\pi, \hat{\gamma}) := S_{i,n-i} + (\hat{\gamma}_k - \hat{\gamma}_{n-i}) \pi_i \hat{\kappa}^{CC}(\pi, \hat{\gamma})
\]

where

\[
\hat{\kappa}^{CC}(\pi, \hat{\gamma}) := \frac{\sum_{j=0}^{n} S_{j,n-j}}{\sum_{j=0}^{n} \hat{\gamma}_{n-j}}
\]

is the Cape Cod loss ratio, which is an estimator of the parameter \( \kappa \). The definition of the Cape Cod predictors reminds us of the identity

\[
E[S_{i,k}] = E[S_{i,n-i}] + (\gamma_k - \gamma_{n-i}) \pi_i \kappa,
\]

which is a consequence of the model assumption. We denote by

\[
\hat{S}^{CC}(\pi, \hat{\gamma}) := (\hat{S}^{CC}_{i,k}(\pi, \hat{\gamma}))_{i,k \in \{0, \ldots, n\}, \ i + k \geq n}
\]

the triangle of all Cape Cod predictors.\(^7\) Letting

\[
\hat{\alpha}^{CC}_i(\pi, \hat{\gamma}) := \pi_i \hat{\kappa}^{CC}(\pi, \hat{\gamma})
\]

the Cape Cod predictors can be written as

\[
\hat{S}^{CC}_{i,k}(\pi, \hat{\gamma}) = S_{i,n-i} + (\hat{\gamma}_k - \hat{\gamma}_{n-i}) \hat{\alpha}^{CC}_i(\pi, \hat{\gamma})
\]

and with

\[
\hat{\alpha}^{CC}(\pi, \hat{\gamma}) := (\hat{\alpha}^{CC}_0(\pi, \hat{\gamma}), \hat{\alpha}^{CC}_1(\pi, \hat{\gamma}), \ldots, \hat{\alpha}^{CC}_n(\pi, \hat{\gamma}))
\]

we obtain

\[
\hat{S}^{CC}(\pi, \hat{\gamma}) = \hat{S}^{BF}(\hat{\gamma}, \hat{\alpha}^{CC}(\pi, \hat{\gamma})).
\]

Therefore, the Cape Cod method with respect to \( \pi \) and \( \hat{\gamma} \) is nothing else than the extended Bornhuetter-Ferguson method with respect to \( \hat{\gamma} \) and \( \hat{\alpha}^{CC}(\pi, \hat{\gamma}) \).

\[\text{We note that the Cape Cod loss ratio } \hat{\kappa}^{CC}(\pi, \hat{\gamma}) \text{ can be written in the form}
\]

\[
\hat{\kappa}^{CC}(\pi, \hat{\gamma}) = \sum_{j=0}^{n} \frac{\hat{\gamma}_{n-j}^{-\pi_j}}{\sum_{h=0}^{j} \hat{\gamma}_{n-h}^{-\pi_h}} S_{j,n-j}
\]

Therefore, the Cape Cod method can be modified by replacing the Cape Cod loss ratio by any other estimator of the form

\[
\hat{\kappa}(\pi, \hat{\gamma}) = \sum_{j=0}^{n} W_j \frac{\hat{S}_{j,n-j}}{\hat{\gamma}_{n-j}^{-\pi_j}}
\]

with random variables (or constants) satisfying \( \sum_{j=0}^{n} W_j = 1 \). For every such modification, the above identities for the Cape Cod predictors remain valid with \( \hat{\alpha}^{CC}_i(\pi, \hat{\gamma}) := \pi_i \hat{\kappa}(\pi, \hat{\gamma}) \) in the place of \( \hat{\alpha}^{CC}_i(\pi, \hat{\gamma}) \) and with

\[
\hat{\alpha}(\pi, \hat{\gamma}) := (\hat{\alpha}_0(\pi, \hat{\gamma}), \hat{\alpha}_1(\pi, \hat{\gamma}), \ldots, \hat{\alpha}_n(\pi, \hat{\gamma}))
\]

in the place of \( \hat{\alpha}^{CC}(\pi, \hat{\gamma}) \).

### 5.6. Additive method

The additive method,\(^8\) which is also called the incremental loss ratio method, is based on the assumption that there exists a vector \( \pi = (\pi_0, \pi_1, \ldots, \pi_n) \) of known volume measures and a vector \( \zeta(\pi) = (\zeta_0(\pi), \zeta_1(\pi), \ldots, \zeta_n(\pi)) \) of parameters such that the identity

\[
E[Z_{i,k}] = \pi_i \zeta_k(\pi)
\]

holds for all \( k \in \{0, 1, \ldots, n\} \) and for all \( i \in \{0, 1, \ldots, n\} \). Then the vector

\[
\zeta(\pi) := (\zeta_0(\pi), \zeta_1(\pi), \ldots, \zeta_n(\pi))
\]

is a development pattern for incremental loss ratios and the vector

\[
\gamma(\pi) := (\gamma_0(\pi), \gamma_1(\pi), \ldots, \gamma_n(\pi))
\]

with

\[
\gamma_k(\pi) = \frac{\sum_{i=0}^{k} \zeta_i(\pi)}{\sum_{i=0}^{n} \zeta_i(\pi)}
\]

as a development pattern for cumulative quotas.

\[\text{An early source containing a description of the Cape Cod method}
\]


\[\text{It is interesting to note that the Cape Cod predictors depend only on the relative size of the volume measures of the different accident years. In fact, for every } c > 0, \text{ we have } \hat{\kappa}^{CC}(c \pi, \hat{\gamma}) = (1/c) \hat{\kappa}^{CC}(\pi, \hat{\gamma}), \text{ and hence } \hat{S}^{CC}(c \pi, \hat{\gamma}) = \hat{S}^{CC}(\pi, \hat{\gamma}).\]

\[\text{The additive method was described by Mack (1997); see also Radtke and Schmidt (2004) where it is pointed out that the predictors of the additive method can be obtained as Gauss-Markov predictors in a suitable linear model.}\]
The additive predictors of the cumulative losses $S_{i,k}$ with $i + k \geq n$ are defined as
\[
\hat{S}_{i,k}^{AD}(\pi) := S_{i,n-i} + \pi_i \sum_{l=n-i+1}^{k} \hat{\zeta}_l^{AD}(\pi)
\]
where
\[
\hat{\zeta}_k^{AD}(\pi) := \frac{\sum_{j=k}^{n-k} Z_{j,k}}{\sum_{j=0}^{n-k} n_j}
\]
is the additive loss ratio introduced in Section 4.
The definition of the additive predictors reminds us of the identity
\[
E[S_{i,k}] = E[S_{i,n-i}] + \pi_i \sum_{l=n-i+1}^{k} \zeta_l(\pi),
\]
which is a consequence of the model assumptions. We denote by
\[
\hat{S}^{AD}(\pi) := (\hat{S}_{i,k}^{AD}(\pi))_{i,k \in \{0, 1, \ldots, n\}}, i + k \geq n
\]
the triangle of all additive predictors. Letting
\[
\hat{\gamma}_k^{AD}(\pi) := \frac{\sum_{l=0}^{k} \hat{\zeta}_l^{AD}(\pi)}{\sum_{l=0}^{n} \zeta_l^{AD}(\pi)}
\]
the additive predictors of the non-observable cumulative losses can be written as
\[
\hat{S}_{i,k}^{AD}(\pi) = S_{i,n-i} + (\hat{\gamma}_k^{AD}(\pi) - \hat{\gamma}_{n-i}^{AD}(\pi))\hat{\alpha}_i^{AD}(\pi)
\]
and with
\[
\hat{\gamma}^{AD}(\pi) := (\hat{\gamma}_0^{AD}(\pi), \hat{\gamma}_1^{AD}(\pi), \ldots, \hat{\gamma}_n^{AD}(\pi))
\]
\[
\hat{\alpha}^{AD}(\pi) := (\hat{\alpha}_0^{AD}(\pi), \hat{\alpha}_1^{AD}(\pi), \ldots, \hat{\alpha}_n^{AD}(\pi))
\]
we obtain
\[
\hat{S}^{AD}(\pi) = \hat{S}^{BF}(\hat{\gamma}^{AD}(\pi), \hat{\alpha}^{AD}(\pi)).
\]

Moreover, it will be shown in Subsection 7.1 that
\[
\hat{\alpha}^{AD}(\pi) = \hat{\alpha}^{CC}(\pi, \hat{\gamma}^{AD}(\pi)).
\]
This yields
\[
\hat{S}^{AD}(\pi) = \hat{S}^{BF}(\hat{\gamma}^{AD}(\pi), \hat{\alpha}^{AD}(\pi))
\]
\[
= \hat{S}^{BF}(\hat{\gamma}^{AD}(\pi), \hat{\alpha}^{CC}(\pi, \hat{\gamma}^{AD}(\pi)))
\]
\[
= \hat{S}^{CC}(\pi, \hat{\gamma}^{AD}(\pi)).
\]

Therefore, the additive method with respect to $\pi$ also coincides with the Cape Cod method with respect to $\pi$ and $\hat{\gamma}^{AD}(\pi)$.

The additive method can be extended by combining the prior estimator $\hat{\alpha}^{AD}(\pi)$ of the expected ultimate losses with an arbitrary prior estimator $\hat{\gamma}$ of the development pattern for cumulative quotas, which results in the version
\[
\hat{S}^{BF}(\hat{\gamma}, \hat{\alpha}^{AD}(\pi))
\]
of the extended Bornhuetter-Ferguson method.

### 5.7. Mack’s method


Mack’s method is based on the assumption of the extended Bornhuetter-Ferguson method and on the assumption that there exists a vector $\pi = (\pi_0, \pi_1, \ldots, \pi_n)$ of known volume measures and a vector $\kappa = (\kappa_0, \kappa_1, \ldots, \kappa_n)$ of parameters such that the identity
\[
\alpha_i = \pi_i \kappa_i
\]
holds for all $i \in \{0, 1, \ldots, n\}$.

Ignoring the possible adjustments due to actuarial judgment that Mack (2006) mentions, Mack’s predictors of the cumulative losses $S_{i,k}$ with $i + k \geq n$ are defined as
\[
\hat{S}^{Mack}_{i,k}(\pi) := S_{i,n-i} + (\hat{\gamma}_k^{Mack}(\pi) - \hat{\gamma}_{n-i}^{Mack}(\pi))\pi_i \kappa_i
\]

---

9It is interesting to note that, just like the Cape Cod predictors, the additive predictors depend only the relative size of the volume measures of the different accident years. In fact, for every $c > 0$, we have $\hat{\gamma}^{AD}(c\pi) = c\hat{\gamma}^{AD}(\pi)$ and $\hat{\alpha}^{AD}(c\pi) = c\hat{\alpha}^{AD}(\pi)$, and hence $\hat{S}^{AD}(c\pi) = c\hat{S}^{AD}(\pi)$.
where
\[
\hat{\gamma}_k^{\text{Mack}}(\pi) := \frac{\sum_{l=0}^{k} \hat{\gamma}_l^{\text{Mack}}(\pi)}{\sum_{l=0}^{k} \hat{\delta}_l^{\text{Mack}}(\pi)}
\]
\[
\hat{\delta}_l^{\text{Mack}}(\pi) := \hat{\delta}_l^{\text{Mack}}(\pi) \sum_{l=0}^{n} \hat{\gamma}_l^{\text{Mack}}(\pi)
\]
and
\[
\hat{\delta}_l^{\text{Mack}}(\pi) := \frac{\sum_{l=0}^{n-i} Z_{i,l}}{\sum_{l=0}^{n-i} \hat{\delta}_l^{\text{AD}}(\pi)}
\]
\[
\hat{\gamma}_k^{\text{Mack}}(\pi) := \frac{\sum_{j=0}^{n-k} Z_{j,k}}{\sum_{j=0}^{n-k} \hat{\gamma}_j^{\text{Mack}}(\pi)}.
\]

We denote by
\[
\hat{S}^{\text{Mack}}(\pi) := (\hat{S}_{i,k}^{\text{Mack}}(\pi))_{i,k \in \{0,\ldots,n\}}, \ i+k \geq n
\]
the triangle of all predictors of Mack’s method.\(^{10}\)

Letting
\[
\hat{\alpha}_i^{\text{Mack}}(\pi) := \pi_i \hat{\gamma}_i^{\text{Mack}}(\pi),
\]
the predictors of Mack’s method can be written as
\[
\hat{S}_{i,k}^{\text{Mack}}(\pi) = S_{i,n-i} + (\hat{\gamma}_k^{\text{Mack}}(\pi) - \hat{\gamma}_{n-i}^{\text{Mack}}(\pi)) \hat{\alpha}_i^{\text{Mack}}(\pi)
\]
and with
\[
\hat{\gamma}_k^{\text{Mack}}(\pi) := (\hat{\gamma}_0^{\text{Mack}}(\pi), \hat{\gamma}_1^{\text{Mack}}(\pi), \ldots, \hat{\gamma}_n^{\text{Mack}}(\pi))
\]
\[
\hat{\alpha}_k^{\text{Mack}}(\pi) := (\hat{\alpha}_0^{\text{Mack}}(\pi), \hat{\alpha}_1^{\text{Mack}}(\pi), \ldots, \hat{\alpha}_n^{\text{Mack}}(\pi))
\]
we obtain
\[
\hat{S}^{\text{Mack}}(\pi) = \hat{S}^{\text{BF}}(\hat{\gamma}^{\text{Mack}}(\pi), \hat{\alpha}^{\text{Mack}}(\pi)).
\]
Therefore, Mack’s method with respect to \(\pi\) is nothing else than the extended Bornhuetter-Ferguson method with respect to \(\hat{\gamma}^{\text{Mack}}(\pi)\) and \(\hat{\alpha}^{\text{Mack}}(\pi)\).

Moreover, it will be shown in Subsection 7.2 that
\[
\hat{\gamma}^{\text{Mack}}(\pi) = \hat{\gamma}^{\text{AD}}(\hat{\alpha}^{\text{LD}}(\hat{\gamma}^{\text{AD}}(\pi)))
\]
\[
\hat{\alpha}^{\text{Mack}}(\pi) = \hat{\alpha}^{\text{AD}}(\hat{\alpha}^{\text{LD}}(\hat{\gamma}^{\text{AD}}(\pi)))
\]
Thus, letting
\[
\hat{\pi}^{\text{Mack}}(\pi) := \hat{\alpha}^{\text{LD}}(\hat{\gamma}^{\text{AD}}(\pi))
\]
we obtain
\[
\hat{\gamma}^{\text{Mack}}(\pi) = \hat{\gamma}^{\text{AD}}(\hat{\pi}^{\text{Mack}}(\pi))
\]
\[
\hat{\alpha}^{\text{Mack}}(\pi) = \hat{\alpha}^{\text{AD}}(\hat{\pi}^{\text{Mack}}(\pi))
\]
and hence
\[
\hat{S}^{\text{Mack}}(\pi) = \hat{S}^{\text{BF}}(\hat{\gamma}^{\text{Mack}}(\pi), \hat{\alpha}^{\text{Mack}}(\pi))
\]
\[
= \hat{S}^{\text{BF}}(\hat{\gamma}^{\text{AD}}(\hat{\pi}^{\text{Mack}}(\pi)), \hat{\alpha}^{\text{AD}}(\hat{\pi}^{\text{Mack}}(\pi)))
\]
\[
= \hat{S}^{\text{AD}}(\hat{\pi}^{\text{Mack}}(\pi)).
\]
This means that Mack’s method consists of two steps:

- First, the initial volume measure \(\pi\) is adjusted via the loss-development method with respect to the additive development pattern \(\hat{\gamma}^{\text{AD}}(\pi)\).
- Second, the ultimate losses are predicted by the additive method with respect to the adjusted volume measure \(\hat{\pi}^{\text{Mack}}(\pi)\).\(^{11}\)

As a consequence of the last result of the previous subsection, we also obtain
\[
\hat{S}^{\text{Mack}}(\pi) = \hat{S}^{\text{CC}}(\hat{\pi}^{\text{Mack}}(\pi), \hat{\gamma}^{\text{AD}}(\hat{\pi}^{\text{Mack}}(\pi)))
\]
which means that Mack’s method with respect to \(\pi\) also coincides with the Cape Cod method with the Bornhuetter-Ferguson method.

5.8. Panning’s method

In response to the CAS 2006 Reserves Call Paper Program, Panning (2006) proposed a quite original method of loss reserving which can also be viewed as another method of parameter estimation for the extended Bornhuetter-Ferguson method.

Panning’s method is based on the assumption that there exists a vector \(\beta = (\beta_0, \beta_1, \ldots, \beta_n)\) of parameters such that the identity
\[
\beta_k = \frac{E[Z_{i,k}]}{E[Z_{i,0}]}
\]
\(^{11}\)Since the adjusted volume measures are loss-development predictors of the ultimate losses, their order of magnitude usually differs from that of the initial volume measures. At first glance, this may cause some irritation, but it is resolved immediately because, as pointed out before, the prior estimators of the additive method depend only on the relative size of the volume measures.
holds for all \(k \in \{0,1,\ldots,n\}\) and for all \(i \in \{0,1,\ldots,n\}\). Then the vector 
\[
\mathbf{\beta} := (\beta_0, \beta_1, \ldots, \beta_n)
\]
is a development pattern for incremental ratios and the vector 
\[
\mathbf{\gamma} := (\gamma_0, \gamma_1, \ldots, \gamma_n)
\]
with 
\[
\gamma_k := \frac{\sum_{l=0}^{k-1} \beta_l}{\sum_{l=0}^{n} \beta_l}
\]
as a development pattern for cumulative quotas. 

Ignoring the adjustment of the Panning ratios of the last development years used by Panning (2006), Panning’s predictors of the cumulative losses \(S_{i,k}\) with \(i+k \geq n\) are defined as 
\[
\hat{S}_{i,k} := S_{i,n-i} + Z_{i,0} \sum_{l=n-i+1}^{k} \hat{\beta}_l
\]
where 
\[
\hat{\beta}_k := \frac{\sum_{j=0}^{n-k} Z_{j,k} Z_{j,0}}{\sum_{j=0}^{n} Z_{j,0}^2}
\]
is the Panning ratio introduced in Section 4. The definition of Panning’s predictors reminds us of the identity 
\[
E[S_{i,k}] = E[S_{i,n-i}] + E[Z_{i,0}] \sum_{l=n-i+1}^{k} \beta_l
\]
which is a consequence of the model assumption. We denote by 
\[
\hat{S}_{i,k}^{\text{Panning}} := \left(\hat{S}_{i,k}\right)_{i,k \in \{0,1,\ldots,n\}, \ i+k \geq n}
\]
the triangle of all predictors of Panning’s method. Letting 
\[
\hat{\gamma}_k := \sum_{l=0}^{n} \frac{\hat{\beta}_l}{\sum_{l=0}^{n} \beta_l}
\]
\[
\hat{\alpha}_i := Z_{i,0} \sum_{l=0}^{n} \frac{\hat{\beta}_l}{\sum_{l=0}^{n} \beta_l}
\]
Panning’s predictors can be written as 
\[
\hat{S}_{i,k}^{\text{Panning}} = S_{i,n-i} + (\hat{\gamma}_k - \hat{\gamma}_{n-i}) \hat{\alpha}_i
\]
and with 
\[
\hat{\gamma}_k := (\hat{\gamma}_0^{\text{Panning}}, \hat{\gamma}_1^{\text{Panning}}, \ldots, \hat{\gamma}_n^{\text{Panning}})
\]
\[
\hat{\alpha}_i := (\hat{\alpha}_0^{\text{Panning}}, \hat{\alpha}_1^{\text{Panning}}, \ldots, \hat{\alpha}_n^{\text{Panning}})
\]
we obtain 
\[
\hat{S}^{\text{Panning}} := \hat{S}^{\text{BF}}(\hat{\gamma}^{\text{Panning}}, \hat{\alpha}^{\text{Panning}}).
\]
Therefore, Panning’s method is noting else than the extended Bornhuetter-Ferguson method with respect to \(\hat{\gamma}^{\text{Panning}}\) and \(\hat{\alpha}^{\text{Panning}}\).

It is remarkable that Panning’s method provides a serious and singular alternative to the chain-ladder method since both methods are entirely based on the (internal) information contained in the run-off triangle.

Panning’s method can be extended in two ways:

An obvious extension of Panning’s method consists of the combination of the prior estimator \(\hat{\alpha}^{\text{Panning}}\) of the expected ultimate losses with an arbitrary prior estimator \(\hat{\gamma}\) of the development pattern for cumulative quotas, which results in the version 
\[
\hat{S}^{\text{BF}}(\hat{\gamma}, \hat{\alpha}^{\text{Panning}})
\]
of the extended Bornhuetter-Ferguson method. This extension of Panning’s method corresponds to the natural extension of the additive method.

Another and slightly less obvious extension of Panning’s method is based on the following observation: Since 
\[
\hat{\beta}_0^{\text{Panning}} = 1
\]
we have 
\[
\hat{\gamma}_0^{\text{Panning}} = \frac{1}{\sum_{l=0}^{n} \hat{\beta}_l^{\text{Panning}}}
\]
and hence 
\[
\hat{\alpha}_i^{\text{Panning}} = \frac{Z_{i,0}}{\hat{\gamma}_0^{\text{Panning}}}.
\]
Thus, for any prior estimator \( \hat{\gamma} \) of the development pattern for cumulative quotas, we may define

\[
\hat{\alpha}_i^{\text{Panning}^*}(\hat{\gamma}) := \frac{Z_{i,0}}{\gamma_0}
\]

(such that \( \hat{\alpha}_i^{\text{Panning}^*}(\hat{\gamma}^{\text{Panning}}) = \hat{\alpha}_i^{\text{Panning}} \)). Letting \( \hat{\alpha}^{\text{Panning}^*}(\hat{\gamma}) := (\hat{\alpha}_0^{\text{Panning}^*}(\hat{\gamma}), \hat{\alpha}_1^{\text{Panning}^*}(\hat{\gamma}), \ldots, \hat{\alpha}_n^{\text{Panning}^*}(\hat{\gamma})) \) we obtain the version

\[
\hat{S}^{\text{BF}}(\hat{\gamma}, \hat{\alpha}^{\text{Panning}^*}(\hat{\gamma}))
\]

of the extended Bornhuetter-Ferguson method. This extension of Panning’s method can be understood as a modification of the loss-development method which is obtained from the latter by replacing the ratios \( S_{i,n-i}/\gamma_{n-i} \) of calendar year \( n \) by the ratios \( Z_{i,0}/\gamma_0 = S_{i,0}/\gamma_0 \) of development year 0.

### 5.9. Remarks

Table 1 compares the different methods of loss reserving considered in this section with regard to the choices of the prior estimator \( \hat{\gamma} \) of the cumulative quotas and the prior estimator \( \hat{\alpha} \) of the expected ultimate losses, respectively. We denote by \( \hat{\gamma}^{\text{external}} \) and \( \hat{\alpha}^{\text{external}} \) any prior estimators of \( \gamma \) and \( \alpha \) which are based on external information and hence yield the Bornhuetter-Ferguson predictors based on external information. Table 1 is to be understood in the sense that, whenever the prior estimators of the expected ultimate losses depend on prior estimators of the cumulative quotas, the prior estimators of the cumulative quotas are also used for the prior estimators of the expected ultimate losses. For example,

- the external loss-development method is the extended Bornhuetter-Ferguson method with respect to the prior estimators \( \hat{\gamma}^{\text{external}} \) and \( \hat{\alpha}^{\text{LD}(\hat{\gamma}^{\text{external}})} \) whereas

- the chain-ladder method is the extended Bornhuetter-Ferguson method with respect to the prior estimators \( \hat{\gamma}^{\text{CL}} \) and \( \hat{\alpha}^{\text{LD}(\hat{\gamma}^{\text{CL}})} \).

The double occurrence of the additive method and of Panning’s method is due to the identities \( \hat{\alpha}^{\text{AD}(\pi)} = \hat{\alpha}^{\text{CC}(\pi, \hat{\gamma}^{\text{AD}(\pi)})} \) and \( \hat{\alpha}^{\text{Panning}} = \hat{\alpha}^{\text{Panning}^*}(\hat{\gamma}^{\text{Panning}}) \).

Table 1 provides a concise and systematic comparison of methods of loss reserving which, to a different degree, are widely used in actuarial practice. Of course, the other combinations of prior estimators of the cumulative quotas and of the expected ultimate losses, which are all distinct and apparently have not been given a name in the literature, could be used as well, and even other choices of prior estimators could be considered. In particular, new prior estimators can be generated by taking convex combinations of the prior estimators given in the table.

In Table 1 we have excluded the prior estimators of the method of Mack which, due to the identities \( \hat{\gamma}^{\text{Mack}(\pi)} = \hat{\gamma}^{\text{AD}(\pi^{\text{Mack}(\pi)})} \) and \( \hat{\alpha}^{\text{Mack}(\pi)} = \hat{\alpha}^{\text{AD}(\pi^{\text{Mack}(\pi)})} \), are special cases of the prior estimators of the additive method with respect to the transformation \( \pi^{\text{Mack}(\pi)} \) of the vol-

---

**Table 1. Comparison of some versions of the extended Bornhuetter-Ferguson method resulting from the use of different prior estimators of the cumulative quotas and the expected ultimate losses**

<table>
<thead>
<tr>
<th>Prior Estimators of Expected Ultimate Losses</th>
<th>( \hat{\gamma}^{\text{external}} )</th>
<th>( \hat{\gamma}^{\text{AD}(\pi)} )</th>
<th>( \hat{\gamma}^{\text{CL}} )</th>
<th>( \hat{\gamma}^{\text{Panning}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \hat{\gamma}^{\text{CC}(\pi, \hat{\gamma})} )</td>
<td>Bornhuetter-Ferguson Method (external)</td>
<td>Additive Method</td>
<td>Chain-Ladder Method</td>
<td>Panning’s Method</td>
</tr>
<tr>
<td>( \hat{\gamma}^{\text{BD}(\pi)} )</td>
<td>Cape Cod Method (external)</td>
<td>Additive Method</td>
<td></td>
<td>Panning’s Method</td>
</tr>
<tr>
<td>( \hat{\gamma}^{\text{Panning}^*(\hat{\gamma})} )</td>
<td>Loss-Development Method (external)</td>
<td>Additive Method</td>
<td></td>
<td>Panning’s Method</td>
</tr>
<tr>
<td>( \hat{\gamma}^{\text{Panning}} )</td>
<td></td>
<td>Additive Method</td>
<td></td>
<td>Panning’s Method</td>
</tr>
</tbody>
</table>

---

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Table 2. Realization of a run-off triangle for cumulative losses, completed by volume measures and realizations of the external prior estimators of the cumulative quotas and the expected ultimate losses

<table>
<thead>
<tr>
<th>Development Year $k$</th>
<th>$\tau_i$</th>
<th>$\hat{\tau}_{\text{external}}^i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.2800</td>
<td>0.2800</td>
</tr>
<tr>
<td>1</td>
<td>0.5300</td>
<td>0.5300</td>
</tr>
<tr>
<td>2</td>
<td>0.7100</td>
<td>0.7100</td>
</tr>
<tr>
<td>3</td>
<td>0.8600</td>
<td>0.8600</td>
</tr>
<tr>
<td>4</td>
<td>0.9500</td>
<td>0.9500</td>
</tr>
<tr>
<td>5</td>
<td>1.0000</td>
<td>1.0000</td>
</tr>
</tbody>
</table>

Table 3. Realizations of the prior estimators of the cumulative quotas

<table>
<thead>
<tr>
<th>Prior Quotas</th>
<th>Development Year $k$</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\hat{\tau}_{\text{external}}^i$</td>
<td>0.2800</td>
<td>0.5300</td>
<td>0.7100</td>
<td>0.8600</td>
<td>0.9500</td>
<td>1.0000</td>
<td></td>
</tr>
<tr>
<td>$\hat{\tau}_{\text{AD}}^i$</td>
<td>0.2626</td>
<td>0.5430</td>
<td>0.7091</td>
<td>0.8623</td>
<td>0.9600</td>
<td>1.0000</td>
<td></td>
</tr>
<tr>
<td>$\hat{\tau}_{\text{CL}}^i$</td>
<td>0.2546</td>
<td>0.5222</td>
<td>0.6939</td>
<td>0.8549</td>
<td>0.9575</td>
<td>1.0000</td>
<td></td>
</tr>
<tr>
<td>$\hat{\tau}_{\text{Panning}}^i$</td>
<td>0.2620</td>
<td>0.5482</td>
<td>0.7137</td>
<td>0.8657</td>
<td>0.9613</td>
<td>1.0000</td>
<td></td>
</tr>
<tr>
<td>$\hat{\tau}_{\text{Mack}}^i$</td>
<td>0.2567</td>
<td>0.5259</td>
<td>0.6970</td>
<td>0.8567</td>
<td>0.9581</td>
<td>1.0000</td>
<td></td>
</tr>
</tbody>
</table>

It is interesting to note that in Mack’s method different development patterns are used for the adjustment of the volume measure and the final application of the additive method with respect to the adjusted volume measure.

In conclusion, the discussion in the present section and, in particular, the above table shows that the extended Bornhuetter-Ferguson method provides a general method under which several methods of loss reserving can be subsumed. The focus on

- prior estimators of the cumulative quotas and
- prior estimators of the expected ultimate losses

provides a large variation of loss reserving methods.

We are thus led to the notion of the Bornhuetter-Ferguson principle. The Bornhuetter-Ferguson principle consists of

- the simultaneous use of various versions of the extended Bornhuetter-Ferguson method,
- the comparison of the resulting predictors, and
- the final selection of best predictors of the ultimate losses.

The Bornhuetter-Ferguson principle should be regarded as a method of loss reserving in its own right which, in every single application, can be specified according to the available sources of information and their degree of credibility and which, in turn, can also be used to check the credibility of these different sources of information.

6. Numerical example

In the present section we present a numerical example to illustrate the possible use of the Bornhuetter-Ferguson principle. Of course, any observations and comments we shall make refer only to the numerical example under consideration, and different data would lead to different observations and conclusions. It is also evident that in actuarial practice a much more refined analysis would be required which, however, could still be performed in the same spirit.

We hope to show that the Bornhuetter-Ferguson principle can be used to select an appropriate version of the extended Bornhuetter-Ferguson method for a given run-off triangle. By contrast, since the selection process is driven by the data and actuarial judgment, it should be clear that the Bornhuetter-Ferguson principle cannot be used to identify a single version which would be superior for every run-off triangle.
Table 4. Realizations of the prior estimators of the expected ultimate losses

<table>
<thead>
<tr>
<th>Accident Year i</th>
<th>Prior Expected Ultimate Losses</th>
<th>Prior Quotas</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>0</td>
</tr>
<tr>
<td>V11</td>
<td>$\hat{\alpha}^\text{external}_i$</td>
<td>3520</td>
</tr>
<tr>
<td>V12</td>
<td>$\hat{\alpha}^\text{mack}_i$</td>
<td>3520</td>
</tr>
<tr>
<td>V13</td>
<td>$\hat{\alpha}^\text{external}_i$</td>
<td>3520</td>
</tr>
<tr>
<td>V14</td>
<td>$\hat{\alpha}^\text{external}_i$</td>
<td>3520</td>
</tr>
<tr>
<td>V21</td>
<td>$\hat{\alpha}^\text{CC}(\pi, \hat{\gamma}^\text{external}_i)$</td>
<td>3703</td>
</tr>
<tr>
<td>V22</td>
<td>$\hat{\alpha}^\text{CC}(\pi, \hat{\gamma}^\text{AD}(\pi))$</td>
<td>3703</td>
</tr>
<tr>
<td>V23</td>
<td>$\hat{\alpha}^\text{CC}(\pi, \hat{\gamma}^\text{CL})$</td>
<td>3760</td>
</tr>
<tr>
<td>V24</td>
<td>$\hat{\alpha}^\text{CC}(\pi, \hat{\gamma}^\text{Panning})$</td>
<td>3690</td>
</tr>
<tr>
<td>V31</td>
<td>$\hat{\alpha}^\text{AD}(\pi)$</td>
<td>3703</td>
</tr>
<tr>
<td>V32</td>
<td>$\hat{\alpha}^\text{AD}(\pi)$</td>
<td>3703</td>
</tr>
<tr>
<td>V33</td>
<td>$\hat{\alpha}^\text{AD}(\pi)$</td>
<td>3703</td>
</tr>
<tr>
<td>V34</td>
<td>$\hat{\alpha}^\text{AD}(\pi)$</td>
<td>3703</td>
</tr>
<tr>
<td>V41</td>
<td>$\hat{\alpha}^\text{CL}(\pi, \hat{\gamma}^\text{external}_i)$</td>
<td>3483</td>
</tr>
<tr>
<td>V42</td>
<td>$\hat{\alpha}^\text{CL}(\pi, \hat{\gamma}^\text{AD}(\pi))$</td>
<td>3483</td>
</tr>
<tr>
<td>V43</td>
<td>$\hat{\alpha}^\text{CL}(\pi, \hat{\gamma}^\text{CL})$</td>
<td>3483</td>
</tr>
<tr>
<td>V44</td>
<td>$\hat{\alpha}^\text{CL}(\pi, \hat{\gamma}^\text{Panning})$</td>
<td>3483</td>
</tr>
<tr>
<td>V51</td>
<td>$\hat{\alpha}^\text{Panning}(\pi, \hat{\gamma}^\text{external}_i)$</td>
<td>3575</td>
</tr>
<tr>
<td>V52</td>
<td>$\hat{\alpha}^\text{Panning}(\pi, \hat{\gamma}^\text{AD}(\pi))$</td>
<td>3813</td>
</tr>
<tr>
<td>V53</td>
<td>$\hat{\alpha}^\text{Panning}(\pi, \hat{\gamma}^\text{CL})$</td>
<td>3932</td>
</tr>
<tr>
<td>V54</td>
<td>$\hat{\alpha}^\text{Panning}(\pi, \hat{\gamma}^\text{Panning})$</td>
<td>3820</td>
</tr>
<tr>
<td>V61</td>
<td>$\hat{\alpha}^\text{Panning}(\pi, \hat{\gamma}^\text{external}_i)$</td>
<td>3820</td>
</tr>
<tr>
<td>V62</td>
<td>$\hat{\alpha}^\text{Panning}(\pi, \hat{\gamma}^\text{AD}(\pi))$</td>
<td>3820</td>
</tr>
<tr>
<td>V63</td>
<td>$\hat{\alpha}^\text{Panning}(\pi, \hat{\gamma}^\text{CL})$</td>
<td>3820</td>
</tr>
<tr>
<td>V64</td>
<td>$\hat{\alpha}^\text{Panning}(\pi, \hat{\gamma}^\text{Panning})$</td>
<td>3820</td>
</tr>
<tr>
<td>V75</td>
<td>$\hat{\gamma}^\text{Mack}_i$</td>
<td>3529</td>
</tr>
</tbody>
</table>

6.1. Data

For $n = 5$, Table 2 contains

- a realization of a run-off triangle of cumulative losses $S_{i,k}$,
- volume measures $\pi_i$ of the accident years,
- realizations of the prior estimators $\hat{\alpha}^\text{external}_i$ the expected ultimate losses, and
- realizations of the prior estimators $\hat{\gamma}^\text{external}_k$ of the cumulative quotas,

where all $\hat{\alpha}^\text{external}_i$ and $\hat{\gamma}^\text{external}_k$ are based on external information. The comparison of the cumulative losses of developments years 0 and 1 indicates that the realization of $S_{4,1}$ could be an outlier, maybe due to a single large claim.

Table 3 displays the realizations of the prior estimators of the cumulative quotas which are used in the different versions of the extended Bornhuetter-Ferguson method. It appears that the development patterns $\hat{\gamma}^\text{CL}$ and $\hat{\gamma}^\text{Mack}_i(\pi)$ are quite similar.

Table 4 displays the realizations of the prior estimators of the expected ultimate losses which are used in the different versions of the extended Bornhuetter-Ferguson method. In Table 4, the additive method and Panning’s method occur twice since $V22 = V32$ and $V54 = V64$. Due to the outlier in accident year 4 and development year 1, the prior estimators of the expected ultimate losses of accident year 4 obtained by the loss development method are extremely high.
Figure 1. Plot of the first-year reserves and the total reserves.

Table 5. Realizations of the first-year reserves and the total reserves

<table>
<thead>
<tr>
<th>Prior Expected Ultimate Losses</th>
<th>Prior Quotas</th>
<th>First-Year Reserve</th>
<th>Total Reserve</th>
</tr>
</thead>
<tbody>
<tr>
<td>V11 (\hat{\alpha}_{\text{external}})</td>
<td>(\hat{\gamma}_{\text{external}})</td>
<td>4164</td>
<td>9964</td>
</tr>
<tr>
<td>V12 (\hat{\alpha}_{\text{external}})</td>
<td>(\hat{\gamma}_{\text{AD}(\pi)})</td>
<td>4284</td>
<td>9948</td>
</tr>
<tr>
<td>V13 (\hat{\alpha}_{\text{external}})</td>
<td>(\hat{\gamma}_{\text{CL}})</td>
<td>4315</td>
<td>10258</td>
</tr>
<tr>
<td>V14 (\hat{\alpha}_{\text{external}})</td>
<td>(\hat{\gamma}_{\text{Panning}})</td>
<td>4295</td>
<td>9872</td>
</tr>
<tr>
<td>V21 (\hat{\alpha}_{\text{CL}(\pi)})</td>
<td>(\hat{\gamma}_{\text{external}})</td>
<td>4530</td>
<td>10973</td>
</tr>
<tr>
<td>V22 (\hat{\alpha}_{\text{CL}(\pi)})</td>
<td>(\hat{\gamma}_{\text{AD}(\pi)})</td>
<td>4687</td>
<td>10976</td>
</tr>
<tr>
<td>V23 (\hat{\alpha}_{\text{CL}(\pi)})</td>
<td>(\hat{\gamma}_{\text{CL}})</td>
<td>4776</td>
<td>11475</td>
</tr>
<tr>
<td>V24 (\hat{\alpha}_{\text{CL}(\pi)})</td>
<td>(\hat{\gamma}_{\text{Panning}})</td>
<td>4687</td>
<td>10859</td>
</tr>
<tr>
<td>V31 (\hat{\alpha}_{\text{AD}(\pi)})</td>
<td>(\hat{\gamma}_{\text{external}})</td>
<td>4531</td>
<td>10974</td>
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<tr>
<td>V33 (\hat{\alpha}_{\text{AD}(\pi)})</td>
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</tr>
<tr>
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<td>10898</td>
</tr>
<tr>
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<td>(\hat{\gamma}_{\text{external}})</td>
<td>4572</td>
<td>11071</td>
</tr>
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<td>V42 (\hat{\alpha}_{\text{LD}(\pi)})</td>
<td>(\hat{\gamma}_{\text{AD}(\pi)})</td>
<td>4770</td>
<td>11279</td>
</tr>
<tr>
<td>V43 (\hat{\alpha}_{\text{LD}(\pi)})</td>
<td>(\hat{\gamma}_{\text{CL}})</td>
<td>4935</td>
<td>11987</td>
</tr>
<tr>
<td>V44 (\hat{\alpha}_{\text{LD}(\pi)})</td>
<td>(\hat{\gamma}_{\text{Panning}})</td>
<td>4769</td>
<td>11159</td>
</tr>
<tr>
<td>V51 (\hat{\alpha}_{\text{Panning}})</td>
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</tr>
<tr>
<td>V52 (\hat{\alpha}_{\text{Panning}})</td>
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<tr>
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<td>4487</td>
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<td>V62 (\hat{\alpha}_{\text{Panning}})</td>
<td>(\hat{\gamma}_{\text{AD}(\pi)})</td>
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<tr>
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<td>(\hat{\gamma}_{\text{CL}})</td>
<td>4651</td>
<td>11141</td>
</tr>
<tr>
<td>V64 (\hat{\alpha}_{\text{Panning}})</td>
<td>(\hat{\gamma}_{\text{Panning}})</td>
<td>4643</td>
<td>10735</td>
</tr>
<tr>
<td>V75 (\hat{\alpha}_{\text{Mack}}(\pi))</td>
<td>(\hat{\gamma}_{\text{Mack}}(\pi))</td>
<td>4851</td>
<td>11706</td>
</tr>
</tbody>
</table>

Minimum: 4164 9872
Maximum: 4935 11987

6.2. Reserves

There are various kinds of reserves which are of interest. The most important ones are perhaps

- the reserves for the different accident years,
- the reserves for the different calendar years,
- the total reserve.

Here we confine ourselves to the first-year reserve (which is the reserve for the first non-observable calendar year) and the total reserve.

Table 5 displays the realizations of the first-year reserves and of the total reserves which are obtained from the data by applying different versions of the extended Bornhuetter-Ferguson method. In Table 5, the additive method and Panning’s method occur twice since V22 = V32 and V54 = V64.

A subset of the pairs of reserves\(^{14}\) presented in Table 5 is plotted in Figure 1. Figure 1 shows that there is a strong positive correlation between the first-year reserves and the total reserves. Moreover, we make the following obser-

\[^{14}\) In order not to overcharge the plot, the pairs of reserves which are based on the prior estimators \(\hat{\alpha}_{\text{AD}(\pi)}\) or \(\hat{\alpha}_{\text{Panning}(\gamma)}\) are omitted.
Figure 2. Plot of the reliable first-year reserves and total reserves.

![Graph showing reliable reserves](image)

- Both reserves are low for the versions V11, V12, V13, V14 using the external prior estimators of the expected ultimate losses.
- Both reserves are relatively low for the versions V11, V21, V41, V61 using the external prior estimators of the quotas.
- Both reserves are relatively high for the versions V13, V23, V43, V63 using the chain-ladder quotas; this is due to the outlier in accident year 4 and development year 1.
- Both reserves are high for the versions V43 (chain-ladder method) and V75 (Mack’s method).

Moreover, there is a high volatility between the pairs of reserves produced by the different versions of the extended Bornhuetter-Ferguson method.

6.3. Reduction to reliable reserves

When combined with actuarial judgement, the previous observations may be used to select predictors providing reliable reserves:

- If the data of the run-off triangle are judged to be highly reliable, then the low predictors which are based of the external prior estimators of the expected ultimate losses and/or the quotas could be discarded.
- Since the predictors produced by the chain-ladder method and by Mack’s method are extremely high, they could be discarded as well.
- The remaining predictors provide ranges for the first-year reserve and for the total reserve which are not too large.

The remaining pairs of reserves could be judged as being reliable and are plotted in Figure 2. Figure 2 shows that the reliable pairs of reserves yield a rather small range for the first-year reserves (about 3% of the maximal value) and a slightly larger range for the total reserves (about 6% of the maximal value).

6.4. Selection of best reserves

Once the reliable reserves are determined, the final problem is to select predictors which can be regarded as best predictors of the ultimate losses. For example, if particularly prudent reserves are required, then one might select the pre-
dictors of version V23 (Cape Cod method with chain-ladder quotas) plotted in Figure 3. Of course, actuarial judgement could also lead to the selection of another version of the extended Bornhuetter-Ferguson method among those which produce reliable reserves.

6.5. Ranges

The selection of a particular version of the extended Bornhuetter-Ferguson method provides predictors which can be regarded as best predictors of the ultimate losses. However, the rules of accounting tend to require not only best predictors but also ranges reflecting the uncertainty of the best predictors.

The Bornhuetter-Ferguson principle also provides an approximate solution to this requirement: since the different versions of the extended Bornhuetter-Ferguson method generate a variety of reserves, they can be used to determine reliable ranges for the ultimate losses. These ranges are, of course, non-probabilistic ones; instead, they reflect the uncertainty caused by the different sources of information used in the different versions of the extended Bornhuetter-Ferguson method.

In our opinion, the ranges provided by the Bornhuetter-Ferguson principle could be more realistic than those obtained from additional and more or less artificial probabilistic assumptions like, e.g., the normal assumption for incremental losses.

6.6. Analysis of the run-off triangle

Beyond the selection of best predictors and ranges, the Bornhuetter-Ferguson principle may also be used to analyze the run-off triangle and hence the portfolio under consideration. We only mention two rather obvious aspects of such an analysis:

- In the case where the predictors based on (external) prior estimators obtained from a market portfolio differ significantly from the other predictors, the structure of the portfolio under consideration is likely to differ from the market portfolio.
- In the case where the predictors based on volume measures differ significantly from the other predictors, there might be something wrong with the volume measures; in particular, if the volume measures are premiums, then the difference between the predictors could indicate inappropriate pricing.
6.7. Refined analysis

The plot presented in Figure 1 is just one of various possibilities in analyzing the effects of the different versions of the extended Bornhuetter-Ferguson method. Other two-dimensional plots could be designed for representing certain pairs of predictors produced by the different versions of the extended Bornhuetter-Ferguson method and could be used for selecting best predictors and ranges or for analyzing the run-off triangle.

7. Proofs

The present section provides proofs of two non-obvious results mentioned in Subsections 5.6 and 5.7, respectively.

7.1. Additive method and Cape Cod method

The following result implies that the additive method with respect to the volume measure \( \pi \) is identical to the Cape Cod method with respect to \( \pi \) and the development pattern \( \hat{\gamma}^{\text{AD}}(\pi) \):

**Lemma** The prior estimators of the additive method satisfy

\[
\hat{\alpha}^{\text{AD}}(\pi) = \hat{\alpha}^{\text{CC}}(\pi, \hat{\gamma}^{\text{AD}}(\pi)).
\]

**Proof** We have

\[
\left( \sum_{l=0}^{n} \hat{\zeta}^{\text{AD}}_l(\pi) \right) \left( \sum_{j=0}^{n} \pi_j \hat{\zeta}^{\text{AD}}_j(\pi) \right) = \sum_{l=0}^{n} \hat{\zeta}^{\text{AD}}_l(\pi) \sum_{j=0}^{n} \pi_j \hat{\zeta}^{\text{AD}}_j(\pi) = \sum_{l=0}^{n} \sum_{j=0}^{n} Z_{j,l} = \sum_{l=0}^{n} S_{l,n-l}.
\]

This yields

\[
\hat{\alpha}^{\text{AD}}_l(\pi) = \pi_l \frac{\sum_{l=0}^{n-i} Z_{i,l}}{\sum_{l=0}^{n-i} \hat{\zeta}^{\text{AD}}_l(\pi)} = \frac{S_{i,n-i}}{\hat{\zeta}^{\text{AD}}_{n-i}(\pi)} \frac{1}{\sum_{l=0}^{n} \hat{\zeta}^{\text{AD}}_l(\pi)} = \hat{\alpha}^{\text{LD}}_i(\hat{\gamma}^{\text{AD}}(\pi)) \]

as was to be shown.

7.2. Mack’s method and additive method

The following result implies that Mack’s method with respect to the volume measure \( \pi \) is identical to the additive method with respect to the (adjusted) volume measure \( \hat{\alpha}^{\text{LD}}(\gamma^{\text{AD}}(\pi)) \):

**Lemma** The prior estimators of Mack’s method satisfy

\[
\hat{\zeta}^{\text{Mack}}(\pi) = \hat{\gamma}^{\text{AD}}(\hat{\alpha}^{\text{LD}}(\gamma^{\text{AD}}(\pi)))
\]

and

\[
\hat{\alpha}^{\text{Mack}}(\pi) = \hat{\alpha}^{\text{AD}}(\hat{\alpha}^{\text{LD}}(\gamma^{\text{AD}}(\pi)))
\]

**Proof** We have

\[
\pi_l \hat{\zeta}^{\text{Mack}}_i(\pi) = \pi_i \frac{\sum_{l=0}^{n-i} Z_{i,l}}{\sum_{l=0}^{n-i} \hat{\zeta}^{\text{AD}}_i(\pi)} = \frac{S_{i,n-i}}{\hat{\zeta}^{\text{AD}}_{n-i}(\pi)} \frac{1}{\sum_{l=0}^{n} \hat{\zeta}^{\text{AD}}_l(\pi)} = \hat{\alpha}^{\text{LD}}_i(\hat{\gamma}^{\text{AD}}(\pi)) \]

as was to be shown.
and hence
\[ \hat{\zeta}_k^{\text{Mack}}(\pi) = \frac{\sum_{j=0}^{n-k} Z_{j,k}}{\sum_{j=0}^{n} \hat{\zeta}_j^{\text{Mack}}(\pi)} \]
\[ = \frac{\sum_{j=0}^{n-k} \hat{\gamma}_j^{\text{LD}}(\hat{\gamma}^{\text{AD}}(\pi))}{\sum_{j=0}^{n} \hat{\zeta}_j^{\text{Mack}}(\pi)} \]
\[ = \hat{\zeta}_k^{\text{AD}}(\hat{\alpha}^{\text{LD}}(\hat{\gamma}^{\text{AD}}(\pi))) \sum_{l=0}^{n} \hat{\zeta}_l^{\text{AD}}(\pi). \]
This yields
\[ \hat{\gamma}_k^{\text{Mack}}(\pi) = \frac{\sum_{l=0}^{n} \hat{\zeta}_l^{\text{Mack}}(\pi)}{\sum_{l=0}^{n} \hat{\zeta}_l^{\text{AD}}(\pi)} \]
\[ = \frac{\sum_{l=0}^{n} \hat{\gamma}_l^{\text{AD}}(\hat{\alpha}^{\text{LD}}(\hat{\gamma}^{\text{AD}}(\pi)))}{\sum_{l=0}^{n} \hat{\zeta}_l^{\text{AD}}(\pi)} \]
\[ = \hat{\gamma}_k^{\text{AD}}(\hat{\alpha}^{\text{LD}}(\hat{\gamma}^{\text{AD}}(\pi))) \]
\[ \text{and} \]
\[ \hat{\alpha}_i^{\text{Mack}}(\pi) = \pi_i \hat{\alpha}_i^{\text{Mack}}(\pi) \]
\[ = \pi_i \hat{\alpha}_i^{\text{AD}}(\pi) \sum_{l=0}^{n} \hat{\zeta}_l^{\text{Mack}}(\pi) \]
\[ = \hat{\alpha}_i^{\text{AD}}(\hat{\gamma}^{\text{AD}}(\pi)) \sum_{l=0}^{n} \hat{\zeta}_l^{\text{AD}}(\hat{\alpha}^{\text{LD}}(\hat{\gamma}^{\text{AD}}(\pi))) \]
\[ = \hat{\alpha}_i^{\text{AD}}(\hat{\alpha}^{\text{LD}}(\hat{\gamma}^{\text{AD}}(\pi))). \]
as was to be shown.

**Acknowledgment**

The authors gratefully acknowledge comments of Axel Reich and of three anonymous referees who helped substantially to clarify the intention of this paper.

**References**


APPENDIX C

LISTING OF VARIOUS OTHER MODELS AND METHODS

From ICBC 2013 Annual Report page 57, which ICBC filed in the RRA is the following; quote:

“3. Critical Accounting Estimates and Judgments
The Corporation makes estimates and judgments that affect the reported amounts of assets and liabilities. These are continually evaluated and based on historical experience and other facts, including expectations of future events that are believed to be reasonable under the circumstances. Management believes its estimates and judgments to be appropriate; however, actual results may be materially different and would be reflected in future periods.

Significant accounting estimates and judgments include:

a) Actuarial methods and assumptions
The Corporation typically employs three standard actuarial methods to analyze the ultimate claims costs:
• The incurred development method;
• The paid development method; and
• The Bornhuetter-Ferguson method.

These methods call for a review of historical loss and count development patterns. As part of this review, the Corporation calculates loss and count development factors, which represent the year-to-year changes in a given accident year’s incurred loss amount. Based on an examination of the loss development factors, the Corporation’s actuaries select their best estimate of development factors that forecast future loss development.

The loss and count development factors rely on a selected baseline. The baseline for the majority of the coverages is the average of the most recent four accident years. The use of a baseline helps maintain consistency in the loss and count development factors from one reserve review to another. Circumstances may arise when the standard methods are no longer appropriate to use. In these cases and in accordance with accepted actuarial practice, modifications to the methods are made or alternative methods are employed that are specific and appropriate to the circumstances. Circumstances may include a change in the claims settlement environment, a change in the handling or reserving of claims, or an emerging trend in the statistical data used in the analysis.”

How much do “YOU” know now?

The following could be what you do not know?

In Exhibit C.0.5 para 19, what is the “Standard Development Method”
also the: “Hindsight Outstanding Severity Method”
In Exhibit C.0.5 para 23, table is the “Actuarial Methods”
In Exhibit C.0.5 para 31, table there are: “MR Actuarial Methods” and the “WB Actuarial Methods”
In Exhibit C.0.5 para 39, table is the “Development by Kind of Loss Method”
In Exhibit C.1.0 para 2, is the “Count Development Method”
In Exhibit C.1.0 para 9, is the “Paid ALAE Development Method”
In Exhibit C.1.0 para 10, is the “Incremental Paid ALAE to Incurred Loss Method”
In Exhibit C.1.3.14 Column (1) is the “Outstanding Loss (Paid Development Method)”
In Exhibit C.1.3.14 Column (1) is the “Outstanding Severity (Paid Development Method)”
In Exhibit C.1.3.16 there are four charts referring to: “Standard Paid Development Method” and the “Selected O/S Method”
In Exhibit C.1.3.18 is the “Incurred Loss Average of Development Methods”
In Exhibit C.1.4.1 are the “Incurred ALAE Summary Development Method” and the “Incurred ALAE Summary Incremental Method”
In Exhibit C.2.0 are the “Incremental Paid Recovery Method” and the “Incremental Paid ALAE Method”
In Exhibit C.3.0 is the “MR Paid Development Method”
In Exhibit C.9.0 is the “Kind of Loss (KOL) Development method”
In Exhibit C.10.0 para 3 Projection – ICBC uses a standard actuarial method, Johnson’s method with alternative weightings suggested by Mango and Allen
In Exhibit G.1 footnotes to Column (i) para 3 “Discounted Cash Flow Method”

If the reader understands the analysis, discussions, the appropriate application, and the integration of all these models, methods as they apply to the Revenue Requirement Application, and is satisfied, my job is done.
If not, then ask questions of the BCUC and ICBC.

The presentation of this evidence is to prompt questions I do not have the skills, insurance business knowledge and actuarial knowledge to investigate, or ask questions.

I must rely on the BCUC Commissioners and staff to affirm the merits, judgement, selections and application of these models and methods in accordance with their “trained knowledge”.

Thank you.
11 Filing Actuary’s Opinion

I, Camille Minogue, am a Fellow of the Casualty Actuarial Society and have prepared the actuarial rate level requirements for Basic Coverage (Plate/Owner Basic and Manual Basic) for the Insurance Corporation of British Columbia.

I CERTIFY THAT:

1. This rate level requirement for Basic Coverage is in respect of all rate class categories of automobile insurance to be effective as of January 1, 2006 for both new business and renewal business.

2. I have reviewed the data underlying this rate level requirement for reasonableness and consistency, and I believe the data is reliable and sufficient for the determination of the indicated changes in average rate level.

3. The indicated changes in average rate level discussed in this application have been calculated in accordance with Accepted Actuarial Practice.

Camille Minogue
Signature of Actuary

22 August 2005
Date
12 Reviewing Actuary’s Opinion

I, William T. Weiland, am a Fellow of the Casualty Actuarial Society and a Fellow of the Canadian Institute of Actuaries. I have reviewed the actuarial rate level requirements for Basic Coverage (Plate/Owner Basic and Manual Basic) prepared by Camille Minogue for the Insurance Corporation of British Columbia as set out in the document “Actuarial Rate Indications Analysis”, dated 22 August 2005.

In my opinion the indicated changes in average rate level presented in the rate application have been calculated in accordance with accepted actuarial practice.

William T. Weiland  
Signature of Actuary  

22 August 2005  
Date
C.12 FILING ACTUARY’S OPINION

89. I, Camille Minogue, am a Fellow of the Casualty Actuarial Society and have prepared the actuarial rate level requirements for Basic Coverage (Plate Owner Basic and Manual Basic) for the Insurance Corporation of British Columbia.

90. I CERTIFY THAT:

1. This rate level requirement for Basic Coverage is in respect of all rate class categories of automobile insurance to be effective as of May 1, 2007 for both new business and renewal business.

2. I have reviewed the data underlying this rate level requirement for reasonableness and consistency, and I believe the data is reliable and sufficient for the determination of the indicated changes in average rate level.

3. The indicated changes in average rate level discussed in this Application have been calculated in accordance with accepted actuarial practice.

Camille Minogue
Signature of Actuary

16 March 2007
Date
C.13 REVIEWING ACTUARY’S OPINION

91. I, William T. Weiland, am a Fellow of the Casualty Actuarial Society and a Fellow of the Canadian Institute of Actuaries. I have reviewed the actuarial rate level requirements for Basic Coverage (Plate Owner Basic and Manual Basic) prepared by Camille Minogue for the Insurance Corporation of British Columbia as set out in the document “Actuarial Rate Level Indication Analysis”, dated 16 March 2007.

92. In my opinion the indicated changes in average rate level presented in the Rate Application have been calculated in accordance with accepted actuarial practice.

William T. Weiland
Signature of Actuary

16 March 2007
Date
D.12 FILING ACTUARY’S OPINION

114. I, Camille Minogue, am a Fellow of the Casualty Actuarial Society and an Affiliate of the Canadian Institute of Actuaries and have prepared the actuarial rate level requirements for Basic Coverage (Plate Owner Basic and Manual Basic) for the Insurance Corporation of British Columbia.

115. I CERTIFY THAT:

1. This rate level requirement for Basic Coverage is in respect of all rate class categories of automobile insurance to be effective as of November 1, 2014 for both new business and renewal business.

2. I have reviewed the data underlying this rate level requirement for reasonableness and consistency, and I believe the data is reliable and sufficient for the determination of the indicated changes in average rate level.

3. The indicated changes in average rate level discussed in this Application have been calculated in accordance with accepted actuarial practice in Canada.

Signature of Actuary: Camille Minogue

Date: 29 August, 2014
D.13 REVIEWING ACTUARY’S OPINION

116. I, William T. Weiland, am a Fellow of the Casualty Actuarial Society and a Fellow of the Canadian Institute of Actuaries. I have reviewed the actuarial rate level requirements for Basic Coverage (Plate Owner Basic and Manual Basic) prepared by Camille Minogue for the Insurance Corporation of British Columbia as set out in the document “Actuarial Rate Level Indication Analysis”, dated 29 August 2014.

117. In my opinion the indicated changes in average rate level presented in the Rate Application have been calculated in accordance with accepted actuarial practice in Canada.

[Signature]

W.T. Weiland
Signature of Actuary

29 August, 2014
Date